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DYNAMICAL PROPERTIES OF AEROSOL BEAMS

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Dynamical Properties of Aerosol Beams

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ABSTRACT

The purpose of these studies was to demonstrate the applicability of the particle beam principle to measure the size of submicron aerosols and to develop and test a theory which describes the particle trajectories in the beam. Particle beams were generated by continuous nozzle expansion. Polystyrene latex spheres ($0.126 \mu\text{m} \leq d_p \leq 1.3 \mu\text{m}$) were utilized as test aerosol. The expansion nozzle was designed to provide a particle flow which is in velocity equilibrium with the air flow inside the nozzle and which leaves the nozzle parallel to its axis. The design was based upon a particle size $d_p \leq 0.7 \mu\text{m}$ and a source pressure $p_0 > 100 \text{ mmHg}$. The nozzle has a total length of 5.4 mm and an exit diameter of 576 μm . The particles were collected on microscope cover glasses which were mounted 2 to 5 nozzle diameters downstream from the nozzle, i. e., at a distance where the particle beam is already separated from the air jet. The deposit is investigated by dark field microscopy. The particles form a disc-like deposit with rather uniform density. Calibration curves are given for the deposit disc diameter in dependence upon the particle size and source pressure. The results show the possibility to determine the size of spherical particulates with an accuracy of about 5% in the tested range $0.126 \mu\text{m} \leq d_p \leq 0.71 \mu\text{m}$. The applicability of the method to determine the size distribution of atmospheric aerosol in the 0.1 to 1 μm region is demonstrated.

Particle trajectories were calculated downstream from the nozzle using the method of characteristics for supersonic flow. Stokes drag was assumed for the interaction between gas flow and particles. The

calculated trajectories show good agreement with the experiment, thus providing for a better understanding and the theoretical design parameters for particle beams. Some results are presented on the beam forming characteristics of capillaries and conical nozzles.

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
INTRODUCTION	1
I. REVIEW OF NOZZLE FLOW	2
A. Subsonic Flow	2
B. Supersonic Flow - Underexpanded Supersonic Jet	9
II. PARTICLE TRAJECTORIES IN THE AIR JET	31
III. DESIGN OF NOZZLE A	37
IV. EXPERIMENTAL PROCEDURE	43
V. EXPERIMENTAL RESULTS AND DISCUSSION	51
A. Nozzle A	51
B. Nozzle B (Capillary)	70
C. Nozzle C (Convergent nozzle with constant cone angle $2\alpha_N = 10.9^\circ$)	74
VI. SUMMARY	78
APPENDICES	
A. COMPUTER PROGRAM FOR SUBSONIC NOZZLE INLET FLOW	80
B. COMPUTER PROGRAM FOR SUPERSONIC UNDER- EXPANDED AIR JET	95
C. COMPUTER PROGRAM FOR PARTICLE TRAJECTORY IN THE UNDEREXPANDED SUPERSONIC AIR JET	101
LIST OF SYMBOLS	120
BIBLIOGRAPHY	122

INTRODUCTION

An aerosol beam is analogous to a molecular beam, a directed almost interaction free flow of aerosol particles. Particle beams are formed by expanding an aerosol into a vacuum chamber where the carrier gas is pumped off. Murphy and Sears (1964) first investigated aerosol beams. Since then properties of particle beams have been studied by Israel and Friedlander (1967), Israel (1969) and Dahneke and Friedlander (1970) and aerosol beams have been applied to measure relative size distributions of KI particles (Selman, 1967) and the distribution of macroion charge to mass ratios (Dole, et al, 1968). Aerosol beams are especially suited for the investigation of particles in the size range from a few hundredths of a micron up to several microns, i. e., for particles which are of great interest due to their impact upon our air environment. Aerosols in this size group, e. g., create much of the atmospheric turbidity and may penetrate into the lung upon inhalation. The possible applications of aerosol beams include size distribution measurements, collection and concentration of aerosol for microchemical analysis, particle coagulation studies and particle surface interaction research.

The purpose of our research was to develop and test an adequate theoretical analysis of the particle beam as an instrument for assessing aerosols and to obtain the parameters that are essential for the design of aerosol beams with special applications.

I. REVIEW OF NOZZLE FLOW

To create a particle beam, an air-particle mixture is expanded through a nozzle into a continuously evacuated chamber. The design is such that the air emerges from the nozzle exit as a sonic jet. The particle content of the flow is less than 10^{-5} gram per gram of air. Therefore, the air flow can be considered unaltered by the presence of the particles. This means that we can consider the flow as one phase flow. Fluid-dynamically we are dealing with subsonic air flow in the source chamber, and inside the nozzle upstream from the nozzle exit. The air flow is supersonic downstream from the nozzle exit.

The theory for the determination of the air flow through a sonic nozzle is presented in the following sections. The mathematical description of the flows are also given.

A. Subsonic Flow

By definition, subsonic flow is a flow with an average velocity or local velocity below the speed of sound waves. Usually the velocity of fluid is expressed dimensionlessly in terms of Mach number which is defined by the ratio of the fluid velocity and its local speed of sound. Mathematically a gas flow can usually be considered incompressible for Mach numbers smaller than 0.8. For larger Mach numbers, however, compressibility has to be taken into consideration. The incompressible, two-dimensional, irrotational, isentropic, subsonic flow can be represented by the following equation,

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} = 0 \quad (1)$$

where η is the stream-function for incompressible flow and x, y are the Cartesian coordinates.

The velocity components are given by

$$\begin{aligned} u_i &= \frac{\partial \eta}{\partial (y/D^*)} \\ v_i &= \frac{-\partial \eta}{\partial (x/D^*)} \end{aligned} \quad (2)$$

with resultant velocity

$$q_i = (u_i^2 + v_i^2)^{1/2}$$

The subscript i denotes incompressible flow; D^* is a characteristic length of the flow system and is set equal to the throat diameter of the nozzle.

The analytical solution for the above second order partial differential equation is very difficult, due to the variety of physical boundaries encountered in actual problems. Therefore, a numerical method known as relaxation method is adopted to solve the problem. (see Shapiro, p. 836)

Let us imagine the x, y -plane in Fig. 1 to be covered by a square mesh with spacing δ . If a certain flow pattern $\eta = \eta(x, y)$ would satisfy Eq. (1), there would be a value of η corresponding to each net point of the meshes, and these η values would be so related as to satisfy Eq. (1) in every small field of flow. This condition may be expressed approximately in finite-difference form by relating the value η_0 at a point 0 to the values of η_1, η_2, η_3 , and η_4 at the surrounding

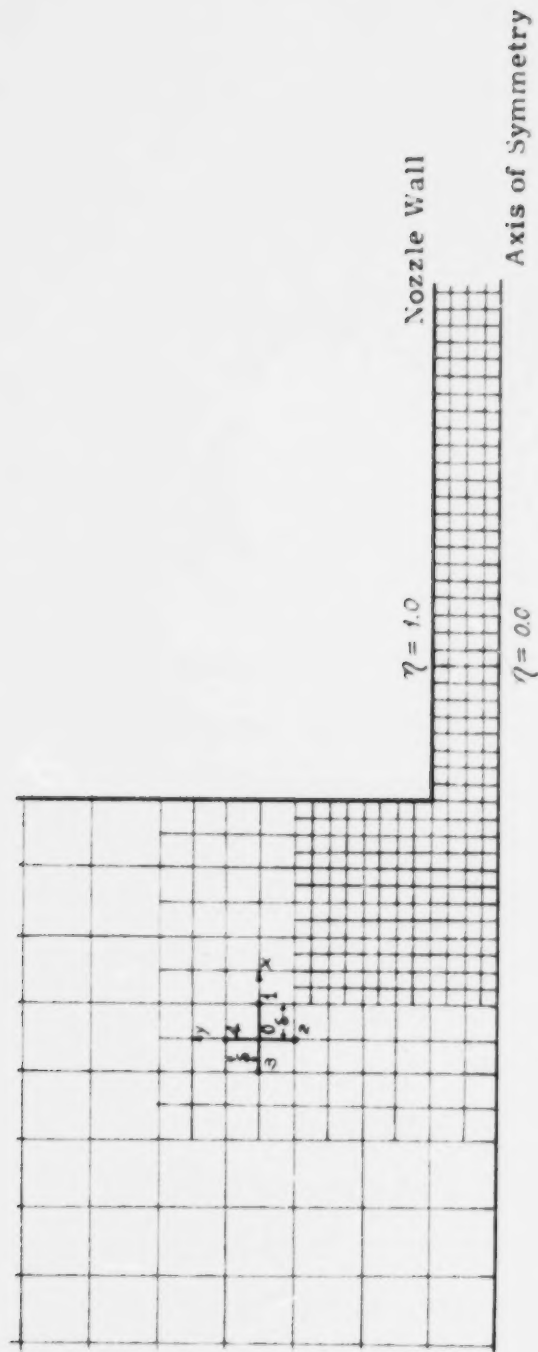


Fig. 1 Relaxation Pattern for the Subsonic Flow Region

net points. The derivatives may be written in the following finite-difference forms:

$$\frac{\partial^2 \eta}{\partial x^2} \approx \frac{1}{\delta} \cdot \left(\frac{\eta_1 - \eta_0}{\delta} - \frac{\eta_0 - \eta_3}{\delta} \right) = \frac{\eta_1 - 2\eta_0 + \eta_3}{\delta^2}$$

$$\frac{\partial^2 \eta}{\partial y^2} \approx \frac{1}{\delta} \cdot \left(\frac{\eta_4 - \eta_0}{\delta} - \frac{\eta_0 - \eta_2}{\delta} \right) = \frac{\eta_4 - 2\eta_0 + \eta_2}{\delta^2}$$

When these are introduced into Eq. (1), we get the finite-difference form of the differential equation as

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \approx \frac{\eta_1 - 2\eta_0 + \eta_3}{\delta^2} + \frac{\eta_4 - 2\eta_0 + \eta_2}{\delta^2} = 0$$

Therefore, we have

$$\eta_1 + \eta_2 + \eta_3 + \eta_4 - 4\eta_0 = 0 \quad (3)$$

To obtain a solution for an incompressible flow past given physical boundaries, the values of η are first guessed at each point of the meshes, based on the known boundary conditions.

Then it is checked whether Eq. (3) is equal to zero or not for any point. If it is not equal to zero, its magnitude is termed the residual. The residual is a measure of the amount by which the differential Eq. (1) is violated at each point with the assumed distribution of η . The residuals are then "relaxed" by systematically adjusting the values of η at each point in such a manner as to reduce the residuals. When all the residuals have been completely relaxed, the distribution of η represents a solution to the Laplace Eq. (1) with the assumed boundary conditions.

In the actual computation, first one releases the center node and fixes the four neighboring nodes, by means of Eq. (3). Then one

calculates the new stream function value for the central node with the aid of the four assumed η values. Subsequently one passes over to the next mesh, and does the same operation for its central node. The computation can be started from the boundaries downward to the axis of symmetry. Thus the boundary information is carried over the whole flow region by this method. In addition, the newly evaluated stream-function values, η , will fit Eq. (3) more closely than the previous assumed values. The result can be refined simply by repeating the procedure in an organized pattern.

The accuracy of this procedure increases, of course, with decreasing mesh size. Since for our problem only the flow near the vicinity of the nozzle inlet is of interest, the finest meshes are taken for this region and coarser meshes are used for the flow region at reasonable distances upstream from the nozzle inlet. The reduction of the mesh spacings is usually done by 50% each time, so that the stream-function values of the new nodes created in the center of the coarser meshes can be evaluated by their four diagonal neighboring nodes with the aid of Eq. (3). Once the first iterated η values of the new nodes of the finer meshes have been assigned, the relaxation process can be applied again and repeated until a satisfactory result is obtained.

Inside the nozzle, the Mach number is always larger than 0.8, i.e., the compressibility of the air is no longer negligible and must be taken into consideration.

The corresponding compressible flow can be described by

$$\frac{\partial}{\partial x} \left(\frac{1}{\rho} \cdot \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{\rho} \cdot \frac{\partial \psi}{\partial y} \right) = 0 \quad (4a)$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial (\ln \rho)}{\partial x} - \frac{\partial \psi}{\partial y} \cdot \frac{\partial (\ln \rho)}{\partial y} = 0 \quad (4b)$$

where ρ is the mass density of the fluid and ψ is the stream function of compressible subsonic flow.

The products of mass density of the fluid and the velocity components are given by

$$\rho \cdot u = \frac{\partial \psi}{\partial y} \quad (5a)$$

$$\rho \cdot v = - \frac{\partial \psi}{\partial x} \quad (5b)$$

$$\rho \cdot q = \left[(\rho \cdot u)^2 + (\rho \cdot v)^2 \right]^{1/2} \quad (5c)$$

If one introduces the dimensionless stream function

$$\psi' = \frac{\psi}{2 \rho_0 C_0 D^2 \rho}$$

with C_0 = sonic speed of fluid at stagnation; subscript o: denoting the state of stagnation, i. e., the state of zero velocity. Eq. (4) and Eq. (5) can be written in finite-difference form as:

$$\begin{aligned} \psi'_1 + \psi'_2 + \psi'_3 + \psi'_4 - 4\psi'_0 \\ - \frac{(\psi'_1 - \psi'_3) \cdot [\ln(\rho/\rho_0)_1 - \ln(\rho/\rho_0)_3]}{4} \\ - \frac{(\psi'_4 - \psi'_2) \cdot [\ln(\rho/\rho_0)_4 - \ln(\rho/\rho_0)_2]}{4} = 0 \end{aligned} \quad (6)$$

and

$$\frac{\rho \cdot q}{\rho_0 C_0} = q_0 \cdot \left[(\psi'_1 - \psi'_3)^2 + (\psi'_4 - \psi'_2)^2 \right]^{1/2} \quad (7)$$

The difference between compressible flow and incompressible flow consists in the last two terms of Eq. (6) which in general do not vary greatly throughout the flow field. A simple analytical method, analogous to the relaxation method described in the preceding paragraphs, was developed by Emman (1944) to solve these equations. The method consists of revising the stream-function values obtained for incompressible flow.

The quantity (ρ/ρ_0) is calculated for each iteration process by a trial-and-error method from Eq. (8) (see Emman, 1944):

$$\frac{\rho \cdot q}{\rho_0 \cdot c_0} = \left(\frac{\rho}{\rho_0} \right) \left[\frac{2}{k-1} \left(1 - \left(\frac{\rho}{\rho_0} \right)^{k-1} \right) \right]^{1/2} \quad (8)$$

where $\frac{\rho \cdot q}{\rho_0 \cdot c_0}$ is given by Eq. (7) and k is the specific heat ratio of the fluid.

The gas velocity, temperature and pressure are the essential parameters that govern the gas-particle interaction. These parameters can be determined from the density ratio ρ/ρ_0 by calculating the Mach number by:

$$M = \sqrt{\frac{2}{k-1} \left[1 - \left(\frac{\rho}{\rho_0} \right)^{k-1} \right]}^{1/2} \quad (9)$$

and subsequently the temperature and pressure ratio in terms of the stagnation state by:

$$\frac{T}{T_0} = \left[1 + \frac{k-1}{2} M^2 \right]^{-1} \quad (10a)$$

$$\frac{P}{P_0} = \left[1 + \frac{k-1}{2} M^2 \right]^{-\frac{k}{k-1}} \quad (10b)$$

$$\frac{\rho}{\rho_0} = \left[1 + \frac{k-1}{2} M^2 \right]^{-\frac{1}{k-1}} \quad (10c)$$

The outlined method was used to determine the air-flow in the vicinity of the inlet of a sharp-edged (90°) capillary of $1000 \mu\text{m}$ length and $170 \mu\text{m}$ inner diameter. Since the flow Reynolds number was estimated to be always smaller than about 2500, Poisseuille flow was assumed upstream from the nozzle inlet and near the capillary outlet. The

dimensionless stream function was chosen $\psi' = 1000$ at the nozzle walls, and $\psi' = 0$ at the axis of symmetry. The calculations were carried out with a UNIVAC 1108. The computer program is given in Appendix A.

The results, i. e., the dimensionless stream-function values, the dimensionless velocity (Mach number) in x-direction, M_x , and in y-direction, M_y , are summarized in Fig. 2 and Fig. 3 and Fig. 4, respectively.

B. Supersonic flow - Underexpanded supersonic jets

In order to calculate the particle trajectories in the gas-particle jet expanding from the nozzle, the aerodynamic properties of the jet must be examined.

The supersonic jet generated by a nozzle in our experiments is underexpanded, i. e., the background pressure in the vacuum chamber is less than the pressure in the nozzle exit plane.

The underexpanded free air jet is a kind of cyclic repetition of expansion and compression processes with shocks as an intermediate product. The typical shape of one cycle of such a jet is shown schematically in Fig. 5. At the beginning of each cycle, the air expands due to the pressure gradient between nozzle exit and chamber background pressure, P_e . The center core of the jet becomes over-rarefied or under-expanded during the expansion process. Subsequently compression takes place near the jet boundary to rise the air pressure to the background pressure, P_e , of the vacuum chamber. The compression in radial directions generates a shock which is called barrel shock due to its shape. The longitudinal compression shock is the Mach disc. The

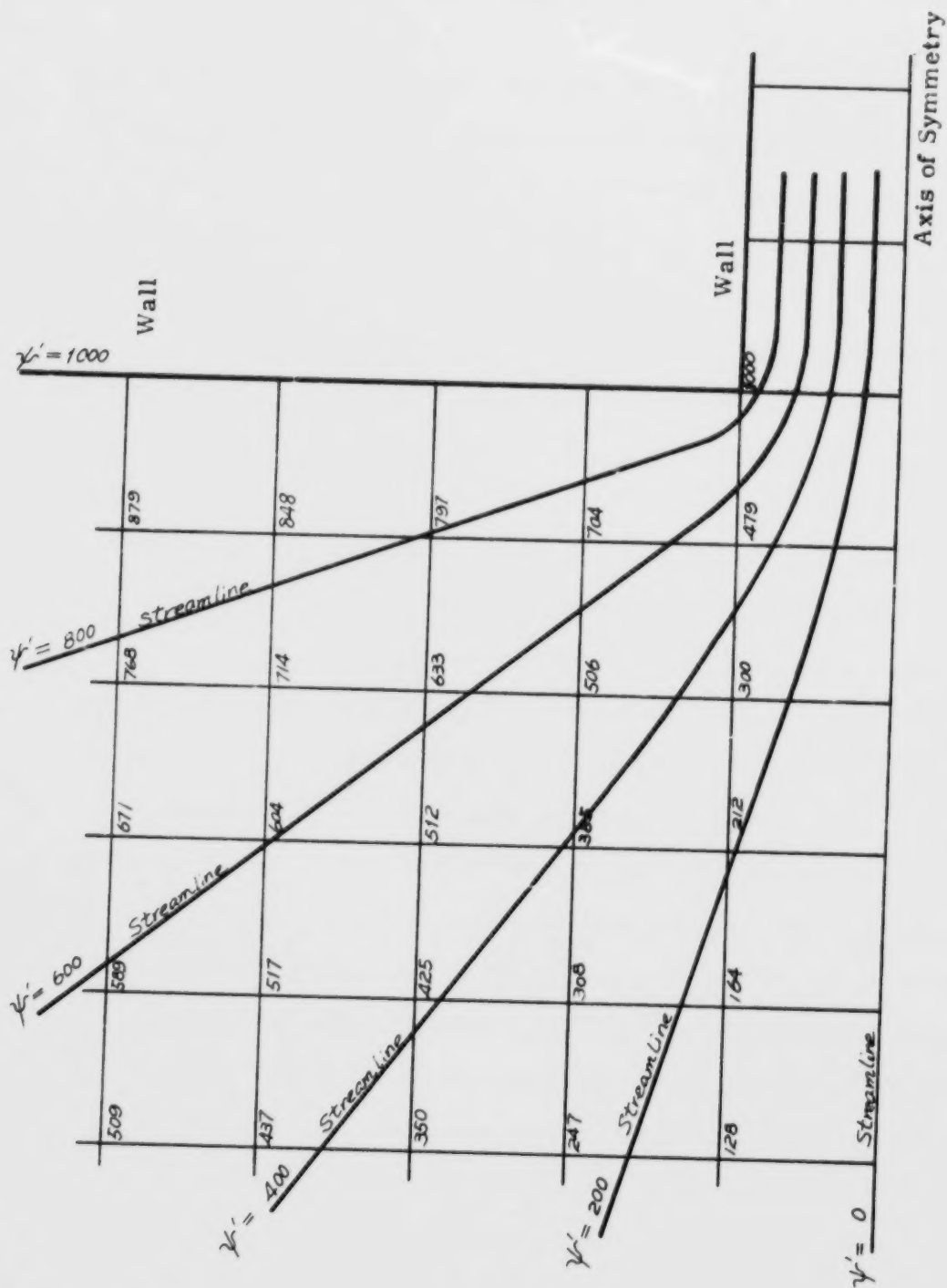


Fig. 2 Dimensionless Stream Function Values, $\psi' \cdot 1000$

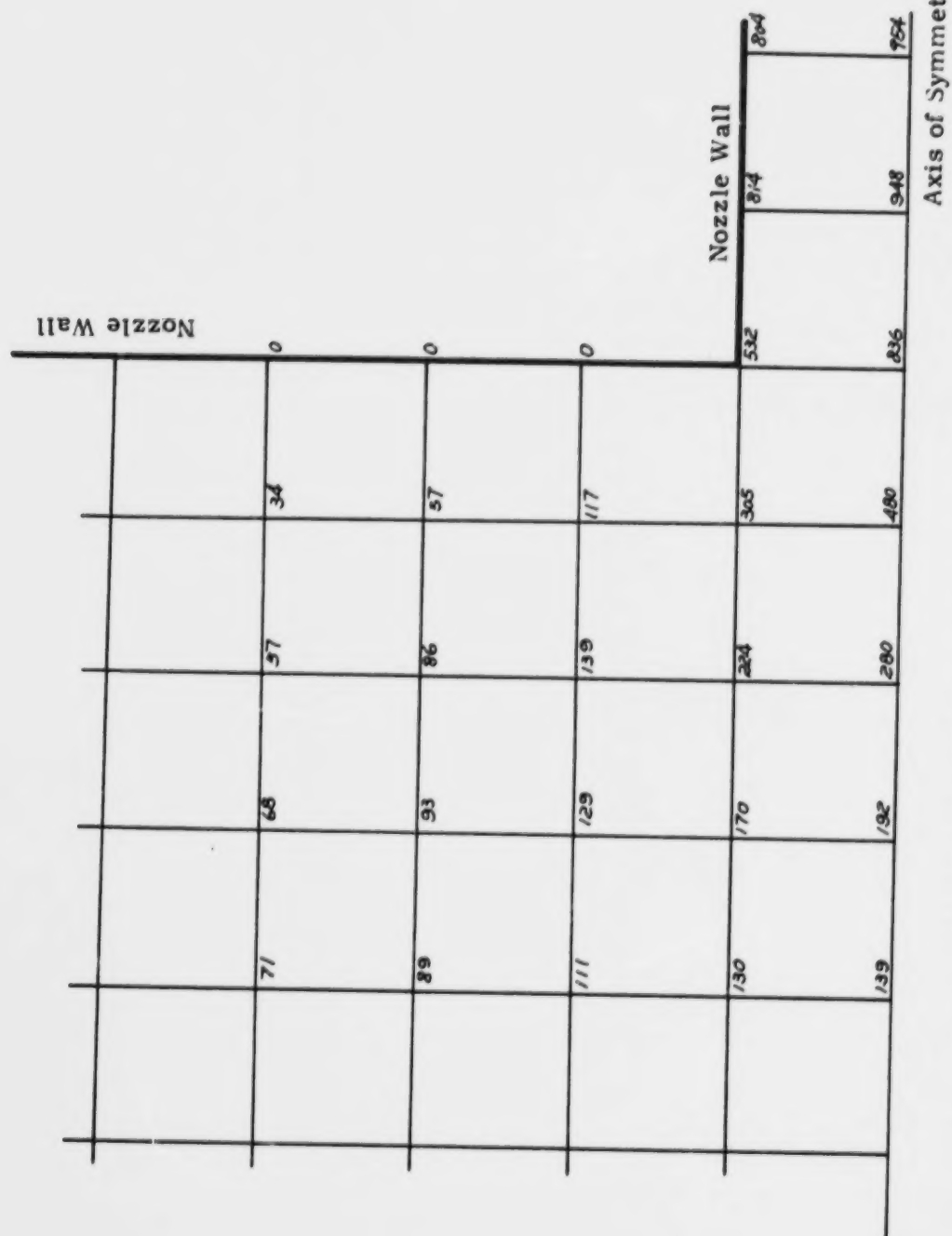


Fig. 3 Dimensionless Velocities in X-Direction, or $M_x \cdot 1000$

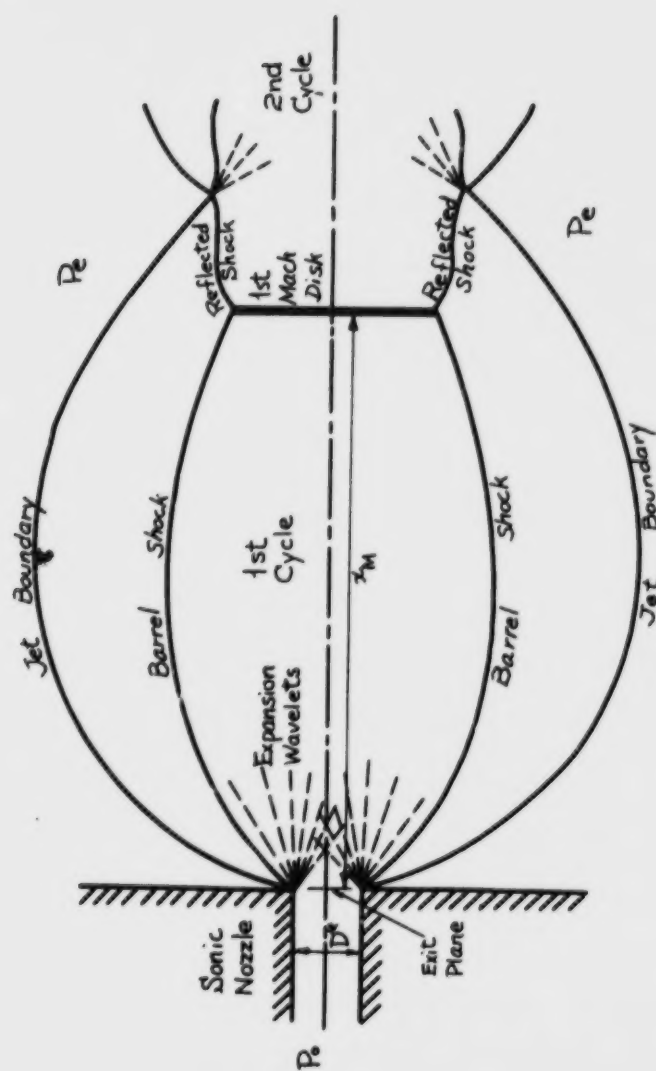


Fig. 5 Underexpanded Supersonic Air Jet
 P_0 = source pressure, P_e = background pressure

strength of these shocks increases with increasing ratio of the source pressure, P_o , to the background pressure, P_e .

The position of the first Mach disc, x_M , is given to very good accuracy by (see e. g. French, 1965)

$$\frac{x_M}{D^*} = 0.67 \cdot (P_o/P_e)^{1/2} \quad (11)$$

where D^* is the throat diameter of the nozzle.

Under our experimental conditions the first Mach disc is located between $10.3 D^*$ and $11.6 D^*$ downstream from the nozzle. Therefore, particles suspended in the air jet have to penetrate one or several Mach discs. However, the thickness of a normal shock is only in the order of the mean free path of the gas molecules, and the particles, which travel with a velocity of several hundred meters per second, penetrate the Mach disc in less than 10^{-5} sec. This time is too short to allow for a measurable momentum exchange between the shock and the particles, i. e., the particle trajectories will not be modified by the shock. Similar considerations hold for the barrel shock. In addition, in most instances the particles will never cross the barrel shock since the expansion angle of a particle beam emerging from the nozzle is always much smaller than the expansion angle of the corresponding air jet.

The physical conditions of a two-dimensional, steady, isentropic, irrotational flow of a perfect gas are expressed mathematically by the differential equation of the complete velocity potential, ϕ .

$$\left[c^2 - \left(\frac{\partial \phi}{\partial x} \right)^2 \right] \frac{\partial^2 \phi}{\partial x^2} - 2 \left(\frac{\partial \phi}{\partial x} \right) \left(\frac{\partial \phi}{\partial y} \right) \left(- \frac{\partial^2 \phi}{\partial x \partial y} \right) + \left[c^2 - \left(\frac{\partial \phi}{\partial y} \right)^2 \right] \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (12)$$

The x- and y-components of the velocity vector, u and v , and the resultant velocity, V , are given by:

$$u = \frac{\partial \phi}{\partial x} ; \quad v = \frac{\partial \phi}{\partial y} ; \quad V = \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right]^{1/2} \quad (13)$$

The local sound velocity, c , is related to the derivatives of the potential by the energy equation,

$$c^2 = c_0^2 - \frac{k-1}{2} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right] \quad (14)$$

where c_0 is the speed of sound at stagnation state and k is the specific heat ratio of the fluid. The differential Eq. (12) is of the general type

$$A \frac{\partial^2 \phi}{\partial x^2} + 2B \frac{\partial^2 \phi}{\partial x \partial y} + C \frac{\partial^2 \phi}{\partial y^2} = D \quad (15)$$

where the coefficients, A, B, C , and D are in general functions of $x, y, \frac{\partial \phi}{\partial x}$, and $\frac{\partial \phi}{\partial y}$.

The method of characteristics can be used to solve Eq. (15) for supersonic flow. The following discussion of this method and its application to the underexpanded free gas jet follows closely the procedure outlined by Shapiro (loc. cit. Vol. I, chap 15).

In this approach, the physical flow plane is divided into many small fields bounded quadrilaterally by Mach-lines or by physical boundaries. All flow properties, e. g., velocity, $\vec{V}$, pressure, P , temperature, T , are considered to be constant in such a field. The small change of the flow properties along the streamline, as it crosses from one field to the adjacent one, is determined by the theory for linearized supersonic flow.

The characteristic curves of Eq. (15) are defined by two pairs of simultaneous, first order, ordinary differential equations:

$$\left(\frac{dy}{dx}\right)_I = \frac{B + \sqrt{B^2 - A \cdot C}}{A}$$

$$\left(\frac{dy}{dx}\right)_{II} = \frac{B - \sqrt{B^2 - A \cdot C}}{A}$$

(16)

$$\left[\frac{d\left(\frac{\partial \phi}{\partial y}\right)}{d\left(\frac{\partial \phi}{\partial x}\right)} \right]_I = - \frac{B + \sqrt{B^2 - A \cdot C}}{C} + \frac{D}{C} \left[\frac{dy}{d\left(\frac{\partial \phi}{\partial x}\right)} \right]_I$$

$$\left[\frac{d\left(\frac{\partial \phi}{\partial y}\right)}{d\left(\frac{\partial \phi}{\partial x}\right)} \right]_{II} = - \frac{B - \sqrt{B^2 - A \cdot C}}{C} + \frac{D}{C} \left[\frac{dy}{d\left(\frac{\partial \phi}{\partial x}\right)} \right]_{II}$$

The subscripts I and II denote characteristic curves of family I and family II respectively.

These characteristic curves satisfy the differential Eq. (15) and therefore may represent solutions of this equation. In addition, the two families of characteristics possess the properties:

- i) Real characteristic curves exist for supersonic, but not for subsonic flow.
- ii) One can allow discontinuities in the derivatives of the fluid properties on the characteristic curve. This means one can patch analytically different flow regions at the characteristic.

For steady isentropic, irrotational flow of a perfect gas, comparison between Eq. (12) and Eq. (15) yields:

$$A = c^2 - u^2 ; \quad B = -u \cdot v ; \quad C = c^2 - v^2 ; \quad D = 0 \quad (17)$$

Substituting these coefficients into Eq. (16) we obtain after some rearrangement:

$$\left(\frac{dy}{dx}\right)_{I,II} = \frac{-\frac{u \cdot v}{c^2} \pm \sqrt{\frac{u^2 + v^2}{c^2} - 1}}{1 - \frac{u^2}{c^2}} \quad (18)$$

$$\left(\frac{dv}{du}\right)_{I,II} = \frac{\frac{u \cdot v}{c^2} \mp \sqrt{\frac{u^2 + v^2}{c^2} - 1}}{1 - \frac{v^2}{c^2}} \quad (19)$$

Eq. (18) defines two characteristic directions at each point of the physical, i. e., x, y -plane. These directions give the slopes of the physical characteristics at each point. Likewise, Eq. (19) defines the slopes of the corresponding pair of hodograph characteristics at the hodograph $(u, v$ -plane) image point of each physical point. These two sets of differential equations taken together permit numerical solution of Eq. (12) to be carried out. For convenience, we introduce the Mach number:

$$M = \sqrt{\frac{u^2 + v^2}{c^2}} = \frac{V}{c} \quad (20)$$

the Mach angle:

$$\alpha = \arctan \frac{1}{\sqrt{M^2 - 1}} \quad (21)$$

and polar coordinates V and Θ for the hodograph plane defined by:

$$u = V \cos \Theta ; \quad v = V \sin \Theta \quad (22)$$

Introducing Eqs. (20), (21), and (22) into Eq. (18), we get after considerable rearrangement:

$$\left(\frac{dy}{dx}\right)_I = \tan(\theta - \alpha)$$

$$\left(\frac{dy}{dx}\right)_{II} = \tan(\theta + \alpha)$$
(23)

Eq. (23) shows that each of the two physical characteristics is inclined at the Mach angle to the velocity vector and that the streamline direction bisects the characteristic directions at that point. This means that the physical characteristics are identical with the Mach lines of the flow. Family I yields the right-running and Family II the left-running Mach waves.

The characteristic curves for any particular problem are found by simultaneous integration of Eqs. (18) and (19). Once these curves are established, the problem is solved for the characteristic curves relate points in the physical plane to points in the hodograph plane, and hence the Mach Number and flow direction may be found for each point of the physical flow field.

The differential equations of the hodograph characteristics, Eq. (19) contain only velocity terms and may therefore be integrated prior to any specific physical problem to give a set of universal hodograph characteristics which are independent of the flow pattern in the physical plane.

For small θ values, Eq. (19) can be approximated, using polar coordinates by:

$$\frac{1}{V} \left(\frac{dV}{d\theta}\right)_{I,II} = \mp \frac{1}{\sqrt{M^2 - 1}} = \mp \tan \alpha$$
(24)

For an adiabatic process dV and dM are related by

$$\frac{dV}{V} = \frac{dM^2}{2M^2(1 + \frac{k-1}{2}M^2)} \quad (25)$$

Hence Eq. (24) may be rewritten in terms of the Mach number

$$(d\theta)_{I,II} = \mp (\sqrt{M^2 - 1} \cdot dM^2) / [2M^2(1 + \frac{k-1}{2}M^2)] \quad (26)$$

Integration of this equation by standard methods yields the hodograph characteristic curves

$$\theta_{I,II} = \mp \sqrt{\frac{k+1}{k-1}} \arctan \sqrt{\frac{k-1}{k+2}(M^2 - 1)} + \arctan \sqrt{M^2 - 1} + \text{constant} \quad (27)$$

The Mach number is very convenient as a dimensionless velocity parameter, but has two disadvantages: (i) it is not proportional to the velocity alone and, (ii) at high speeds it tends toward infinity. A convenient reference velocity is the critical speed, c^* , i. e., the velocity at Mach number one. Using an asterisk to denote conditions at $M = 1$, we have

$$M^* = \frac{V}{c^*} \quad (28)$$

Using the adiabatic relation that

$$M^2 = \frac{2}{k+1} M^{*2} / \left[1 + \frac{k-1}{k+1} M^{*2} \right] \quad (29)$$

the hodograph characteristic curves can be written in the form:

$$\theta_I = -\omega(M^*) + 2K_I - 1000 \quad (30a)$$

$$\theta_{II} = +\omega(M^*) + 2K_{II} - 1000 \quad (30b)$$

where the function $\omega(M^*)$ is given in radian by

$$\omega(M^*) = \sqrt{\frac{k+1}{k-1}} \arctan \sqrt{\frac{M^{*2}-1}{\frac{k+1}{k-1}-M^{*2}}} - \arctan \sqrt{\frac{M^{*2}-1}{1-\frac{k-1}{k+1}M^{*2}}} \quad (31)$$

Since the Mach number, M , and therewith, M^* uniquely determines the flow properties like pressure ratio, temperature ratio, etc., (see Eq. (10)) for isentropic flow, it follows that ω is also directly related to these flow parameters. The constant of integration, denoted by $(2K_{I, II} - 1000)$ in Eq. (30) is determined from the initial values of and M^* for the particular problem.

In Eq. (30 a) the subscript I signifies that pressure waves of family I are absent, that the process occurs at a constant value of K_I , and that changes in flow properties take place as the streamline crosses Mach waves of family II, i.e., as it crosses left-running Mach waves. The meaning of subscript II can be formulated similarly.

In the presence of waves of both families, a simple stepwise method may be used to solve the problem numerically. This procedure is based on the physical fact that changes in fluid properties in a supersonic flow may be conceived as being brought about by waves propagated along the Mach lines of the flow.

The underlying idea is to imagine, for purposes of calculation,

that the physical flow plane is divided into many small regions, in each of which all flow properties, including velocity, direction, pressure, etc., are considered constant. Since the fluid properties change only slightly from one region to the adjacent one, Eq. (12) may be linearized and the linearized theory for supersonic flow with small perturbations (Shapiro loc. cit., Chap 14) may be applied to calculate the changes in fluid properties as a streamline crosses from one field to the next. The linear theory shows that the changes of properties occur as the streamline crosses a Mach line, hence it follows that in general each region is quadrilateral bounded either by Mach lines or physical boundaries. Occasionally the Mach quadrilateral takes on the limiting form of a triangle, as will be seen in the subsequent example.

The numerical procedure may be formulated as follows: The Mach net is built up stepwise by repetitive use of an elementary step in which the fluid properties and location of two initial points establish the fluid properties and location at a third point lying at the intersection of the Mach waves passing through the two initial points.

The procedure is demonstrated in Fig. 6. Points 1, 2, and 3 represent three different fields adjacent to each other. Assume that we know $\omega_1, \theta_1, x_1, y_1$ and $\omega_2, \theta_2, x_2, y_2$, i. e., all the fluid properties of fields 1 and 2 and the location of fields 1 and 2. Field 3 be the intersection between the I-characteristic through point 1 and the II-characteristic through point 2. The curved Mach lines are replaced by a set of straight line chords between the points of the nets in this stepwise procedure.

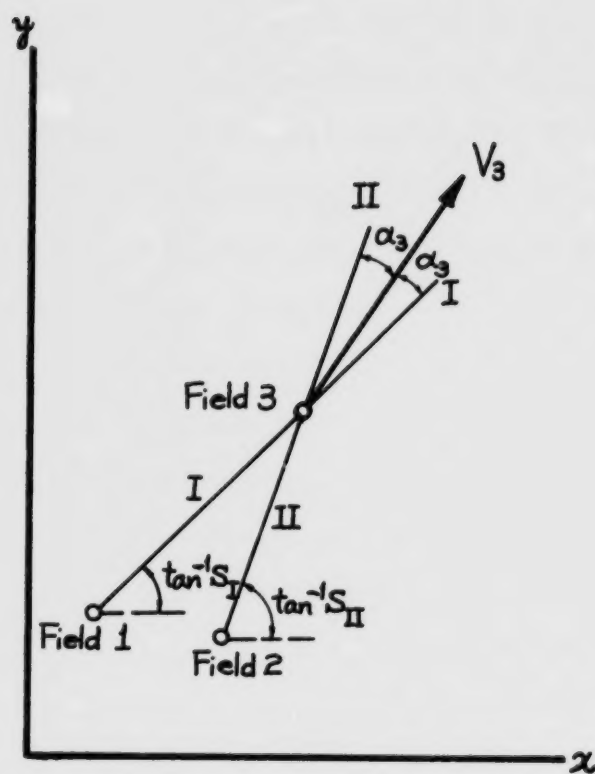


Fig. 6 Unit Process of Characteristics Method

Let S_I be the average slope of the chord 1-3 which approximates the Mach line 1-3; and, similarly, let S_{II} be the slope of the Mach line 2-3.

S_I and S_{II} are then given, according to Eq. (23), approximately by

$$\begin{aligned} S_I &= \tan \frac{1}{2} [(\theta_3 - \alpha_3) + (\theta_1 - \alpha_1)] = \tan (\overline{\theta - \alpha})_{1-3} \\ S_{II} &= \tan \frac{1}{2} [(\theta_3 + \alpha_3) + (\theta_2 + \alpha_2)] = \tan (\overline{\theta + \alpha})_{2-3} \end{aligned} \quad (32)$$

For a given point in the hodograph plane, Eq. (30) may be written:

$$2K_I = \theta + \omega + 1000 \quad (33a)$$

$$2K_{II} = \theta - \omega + 1000 \quad (33b)$$

By adding and subtracting this pair of simultaneous equations, we get

$$\theta = K_I + K_{II} - 1000 \quad (34a)$$

$$\omega = K_I - K_{II} \quad (34b)$$

Thus K_I and K_{II} can be calculated for fields 1 and 2 by Eq. (34). The values $(K_I)_3$ and $(K_{II})_3$ of point 3 are found by virtue of the definition of point 3 by

$$(K_I)_3 = (K_I)_1$$

and

$$(K_{II})_3 = (K_{II})_2$$

Having found $(K_I)_3$ and $(K_{II})_3$ Eq. (34) may be used to compute the fluid properties ω_3 and θ_3 at point 3 and subsequently α_3 from Eqs. (31), (29) and (21). Introducing θ_3 and α_3 into Eq. (32) allows

to calculate the slopes S_I and S_{II} of the Mach lines.

The physical location of point 3 with the coordinates (x_3, y_3) must satisfy the equations:

$$\begin{aligned} y_3 - y_1 &= S_I(x_3 - x_1) \\ y_3 - y_2 &= S_{II}(x_3 - x_2) \end{aligned} \quad (35)$$

Solving for x_3 and y_3 yields the coordinates of point 3:

$$\begin{aligned} x_3 &= \frac{(y_2 - S_{II} \cdot x_2) - (y_1 - S_I \cdot x_1)}{S_I - S_{II}} \\ y_3 &= \frac{S_I(y_2 - S_{II} \cdot x_2) - S_{II}(y_1 - S_I \cdot x_1)}{S_I - S_{II}} \end{aligned} \quad (36)$$

Illustration of the Method

Let us consider the underexpanded air flow at the exit of a two-dimensional nozzle. From symmetry consideration, the middle streamline must be straight, and may be replaced by a wall, thus dividing the flow into two symmetric sections. In the following we consider only one of these sections as shown in Fig. 7.

Let us assume the air emerges from the nozzle parallel to its axis with sonic velocity. This means $M_j = 1$ and $\theta_j = 0$ if the subscript j refers to the fluid properties in the nozzle exit plane. Introducing $M_j = 1$ into Eqs. (10) and (31) yields P_j and ω_j . $(K_I)_j$ and $(K_{II})_j$ are then computed from Eq. (33) with the known ω_j and θ_j values. Thus we find for the first field, designated as (1, 1) in Fig. 7:

$$M(1, 1) = M_j = 1.0; \quad \theta(1, 1) = \theta_j = 0; \quad \omega(1, 1) = \omega_j = 0;$$

$$K_I(1, 1) = (K_I)_j = 500 \quad \text{and} \quad K_{II}(1, 1) = (K_{II})_j = 500.$$

Suppose that the background pressure P_e is 30 mmHg, and source pressure P_o is 760 mmHg. Since the boundary of the free jet must be everywhere at a pressure of 30 mmHg, the sudden pressure drop from 760 mmHg to 30 mmHg at the corner of the nozzle can only take place in an expansive corner type flow.

If field (1, 6) is the last state of the corner flow in Fig. 7, we have

$$P_e / P_o = P(1, 6) / P_o = 30 \text{ mmHg} / 760 \text{ mmHg} = 0.0392$$

Introducing the pressure ratio in Eq. (10) we can compute $M(1, 6)$ and subsequently $M^*(1, 6)$ and $\omega(1, 6)$ from Eqs. (29) and (31). Since fields (1, 6) and (1, 1) lie on a common I-characteristic, we have $K_I(1, 6) = K_I(1, 1) = 500$. Hence, Eqs. (33a) and (33b) yield $\theta(1, 6) = -45^\circ$ and $K_{II}(1, 6) = 455$. Therefore, there is a 45° deflection of the jet boundary as it leaves the nozzle.

All the details of the corner flow between fields (1, 1) and (1, 6) are known analytically from the simple-wave formulas, i. e., Eq. (30a) and Eq. (10). If we divide, e. g., the corner flow between (1, 1) and (1, 6) into six fields, every successive field provides for a 9° deflection of the streamline. Therefore the θ values corresponding to fields (1, 1) to (1, 6) are known. Again, flow fields (1, 1) to (1, 6) lie on common I-characteristic, which means $K_I(1, n) = 500$, for $n = 1, 2, 3, 4, 5, 6$. Thus $\omega(1, n)$, and $K_{II}(1, n)$ can be determined from Eq. (33a). The M^* value can be calculated by trial-and-error method from Eq. (31) with the aid

of the $\omega(1, n)$ values. The corresponding properties, e.g., temperature ratio, pressure ratio, density ratio and Mach number can be calculated by Eq. (10).

Subsequently, when waves of both families are present, the above discussed stepwise method is adopted. For the convenience of illustration, we designate the Mach waves, which come from the corner of the nozzle, a, b, c, d, e, etc., (see Fig. 7).

When wave 'a' strikes the upper wall, it must be reflected as wave 'f' in order to have the flow follow the upper wall (or corner). If the region downstream from wave 'f' is called field (2, 1), we have

$\theta(2, 1) = 0^\circ$ and $K_{II}(2, 1) = K_{II}(1, 2) = 491$, since field (2, 1) lies on the same II-characteristic as field (1, 2). As a result, one finds $K_I(2, 1) = 509$ and $\omega(2, 1) = 27^\circ$ from Eq. (33). Now, assume that wave 'b' is continued as wave 'k', and that wave 'f' is continued as wave 'g', and that they partially bound field (2, 2). But field (2, 2) is reached from field (2, 1) along a characteristic of family II. Therefore, $K_I(2, 2) = K_I(2, 1) = 509$ and $K_{II}(2, 2) = K_{II}(1, 2) = 482$, which yields $\omega(2, 2) = 27^\circ$, and $\theta(2, 2) = -9^\circ$.

The same calculation procedure can be carried out for the rest of the fields except for the regions which form the free boundary of the jet. Since air has been rarefied during the expansion process, the pressure in field (2, 5) is lower than the background pressure. Therefore, 'j' must be reflected in the form of wave 'o' to preserve the boundary condition of constant background pressure at the jet boundary. If the field (2, 6) downstream of wave 'o' is in equilibrium with the background pressure, we have $\omega(2, 6) = \omega(1, 6) = 45^\circ$ and

$K_I(2, 6) = K_I(2, 5) = 509$ which results in $K_{II}(2, 6) = 464$ and $\theta(2, 6) = -27^\circ$. The lower side of region (2, 6) is the jet boundary.

The various fluid properties of each field are summarized in Table 1. For each field, two of the four properties, K_I , K_{II} , ω , and θ are underlined, thereby indicating the two properties in that field which are found first. The remaining two properties in each field are, of course, found from the two underlined ones algebraically from Eqs. (33) and (34).

Table 2 is self-explanatory and shows for each field the direction for the left-running waves, $(\theta + \alpha)$, and for the right-running waves, $(\theta - \alpha)$. As mentioned earlier, in applying the linear theory to each Mach wave, we employ the mean Mach angle to the mean flow direction instead of the Mach angle decided by either the upstream field or downstream field. The mean $(\theta + \alpha)$ values for family I and the mean $(\theta - \alpha)$ values for family II are tabulated in Table 2.

The coordinates of the points of intersection of the Mach waves are determined stepwise by Eq. (36) using the θ -values of Table 1 and the mean Mach angles of Table 2 to calculate S_I and S_{II} from Eq. (32).

The actual calculations were carried out with a Univac 1108 computer. The flow was subdivided into nets with a 1-degree deflection of the streamline. The computer program is given in Appendix B.

Table 1

Calculation Result of Underexpanded Supersonic Air Jet by
Method of Characteristics with Initial Conditions: $M_j=1.00$,
 $P_o=760\text{mmHg}$, $P_e=30\text{mmHg}$, and $\theta_j=0^\circ$

== calculated by boundary
conditions

— calculated by characteristics
method

Field No.	K_I	K_{II}	ω (deg.)	θ (deg.)	α (deg.)	$\theta+\alpha$ (deg.)	$\theta-\alpha$ (deg.)	P/P_o	M	s/s_o
(1, 1)	500	500	<u>0</u>	<u>0</u>	90	90	-90	<u>0.528</u>	<u>1.01</u>	0.634
(1, 2)	<u>500</u>	491	9	<u>-9</u>	45.6	36.6	-54.6	0.314	1.40	0.437
(1, 3)	<u>500</u>	482	18	<u>-18</u>	35.9	17.9	-53.9	0.1196	1.71	0.316
(1, 4)	<u>500</u>	473	27	<u>-27</u>	29.6	2.6	-56.6	0.124	2.02	0.225
(1, 5)	<u>500</u>	464	36	<u>-36</u>	25	-12	-61	0.072	2.37	0.152
(1, 6)	<u>500</u>	455	<u>45</u>	-45	2.12	-23.8	-66.2	<u>0.0392</u>	<u>2.76</u>	0.099
(2, 1)	509	<u>491</u>	18	<u>0</u>	35.9	35.9	-35.9	0.1196	1.71	0.316
(2, 2)	<u>509</u>	<u>482</u>	27	-9	29.6	20.6	-38.6	0.124	1.71	0.225
(2, 3)	<u>509</u>	<u>473</u>	36	-18	25	7	-43	0.0717	2.37	0.152
(2, 4)	<u>509</u>	<u>464</u>	<u>45</u>	-27	21.2	-5.8	-48.2	0.0392	2.76	0.099
(2, 5)	<u>509</u>	<u>455</u>	54	-36	18.0	-28	-54	0.0195	3.23	0.059
(2, 6)	<u>509</u>	464	<u>45</u>	-27	21.2	-5.8	-48.2	<u>0.0392</u>	<u>2.76</u>	0.099
(3, 1)	518	<u>482</u>	36	<u>0</u>	25	25	-25	0.0717	2.37	0.152
(3, 2)	<u>518</u>	<u>473</u>	45	-9	21.2	12.2	-30.2	0.0392	2.76	0.099
(3, 3)	<u>518</u>	<u>464</u>	54	-18	18.0	0	-36	0.0195	3.23	0.059
(3, 4)	<u>518</u>	<u>455</u>	63	-27	15.3	-12.3	-42.3	0.0086	3.8	0.034
(3, 5)	<u>518</u>	<u>464</u>	54	-18	18.0	0	-36	0.0195	3.23	0.059
(3, 6)	<u>518</u>	473	<u>45</u>	-9	21.2	12.2	-30.2	<u>0.0392</u>	<u>2.76</u>	0.099

Table 2

The Angle of Inclination of Mach Waves

Wave	$(\theta + \alpha)$				$(\theta - \alpha)$			
	Up-St	Dn-St	Mean		Up-St	Dn-St	Mean	
	degr.	deg	deg		deg	deg	deg	
a	90	36.6	63.3					
b	36.6	17.9	27.3					
c	17.9	2.6	10.3					
d	2.6	-12	-4.7					
e	-12	-23.8	-17.9					
f					-54.6	-35.9	-40.3	
g					-53.9	-38.6	-46.3	
h					-56.6	-43	-54.8	
i					-61	-48.2	-54.6	
j					-66.2	-54	-60.1	
k	35.9	20.6	28.3					
l	20.6	7	13.8					
m	7	-5.8	0.6					
n	-5.8	-28	-16.9					
o	-28	-5.8	-16.9					
p					38.6	-25	-31.8	
q					-43	-30.2	-36.6	
r					-48.2	-36	-42.1	
s					-54	-42.3	-48.2	
t					-48.2	-36	-42.1	

II. PARTICLE TRAJECTORIES IN THE AIR JET

It was one of the main objectives of this investigation to develop a numerical method that allows to predict the properties of particle beams and to test this method experimentally.

An aerosol beam is in essence characterized by the velocity and trajectory of the particles travelling in the beam. These parameters are determined by the interaction of the gas flow with the particles in the source chamber near the nozzle inlet, inside the nozzle and in the measuring chamber downstream from the nozzle. The interaction consists almost exclusively of drag forces with a certain lag or slippage of the particles in respect to the expanding gas jet.

Since the gas flow can be considered unaltered by the presence of particles, the problem can be solved in two independent steps. First, one determines the flow field in the various sections by the methods outlined in the preceeding chapter. In the second step the particles are added to the known gas flow and their trajectories calculated.

To simplify the mathematical treatment of the problem, we will consider first the region downstream from the nozzle, i. e., the region of supersonic flow. The following numerical model for the particle trajectories is based upon the assumptions:

1. Particles do not interact and the gas flow is undisturbed by the presence of the particles.
2. Brownian motion of the particles can be neglected.
3. The particles are spherical and monodisperse.

4. Drag forces are the only forces acting to accelerate the particles and can be described by Stokes law with appropriate Cunningham correction factor.
5. The gas is inviscid except for the drag it exerts on the particles.
6. The gas may be considered perfect.
7. The gas flow is two-dimensional, irrotational, steady and isentropic.
8. The nozzle exit has a sharp 90 degree corner.
9. In the nozzle exit plane the gas flow is uniform, parallel to the nozzle axis and has the Mach number $M = 1.0$.
10. The particle velocity is in equilibrium with the air flow at the nozzle exit.

The solution for the underexpanded air jet was presented in the preceeding chapter. The particle trajectories can be calculated as follows:

The motion of a particle in a viscous medium in the absence of external forces can be described for small Reynolds numbers by

$$\frac{d\vec{V}_p}{dt} = + \frac{\vec{V}_g - \vec{V}_p}{C \tau} \quad (37)$$

with

$$\tau = \frac{\rho_p d_p^2}{18 \eta_g} \quad (38)$$

and the Cunningham correction factor

$$C = 1.0 + \frac{2 \lambda}{d_p} \left[1.20 + 0.4 \exp \left(-\frac{0.45 d_p}{\lambda} \right) \right] \quad (39)$$

as given by McFarland and Zeller (1963).

$\vec{V}_p$ and $\vec{V}_g$ are the particle and gas velocity, d_p and ρ_p are particle diameter and mass density; the gas viscosity η_g is a function of the gas temperature and is tabulated for air, e.g., in CRC Handbook (46th ed., 1965), λ is the mean free path of the gas molecules which can be calculated for air according to U.S. Weather Bureau ICAO Tables from

$$\lambda = 1.75 \cdot 10^{-5} \left[\frac{T (^{\circ}\text{K})}{P (\text{mmHg})} \right] (\text{cm}) \quad (40)$$

The particle trajectories in the jet can be calculated in the following manner. Assume that a particle enters a field of known and constant flow properties at the given point $P_1(x_1, y_1)$ with the known velocity $\vec{V}_p(1)$, the particle leaves the field at $P_2(x_2, y_2)$ with $\vec{V}_p(2)$, which are unknowns. The particle trajectory in this field can be determined by the following procedure.

We assume that the particles cross the field on a rectilinear path with the average velocity $(\vec{V}_p)_{av}$ and that Eq. (37) may be used in its difference form to describe the particle motion within the field. These assumptions are justified since a field of constant flow properties constitutes only a very small area of the whole flow field. The particle motion in the field is described by:

$$\Delta \vec{V}_p = \frac{1}{C\tau} \cdot \left[\vec{V}_g - (\vec{V}_p)_{av} \right] \cdot \Delta t \quad (41)$$

$$\text{with} \quad (\vec{V}_p)_{av} = \frac{\vec{V}_p(1) + \vec{V}_p(2)}{2} \quad (42)$$

The travel time of the particle, Δt , is given by

$$\Delta t = \frac{x_2 - x_1}{(V_{px})_{av}} = \frac{y_2 - y_1}{(V_{py})_{av}} \quad (43)$$

The average velocity $(\vec{V}_p)_{av}$, P_2 and $\vec{V}_p(2)$ can be calculated from Eq. (41) through an iteration process.

One starts with the assumption $\vec{V}_p(2) = \vec{V}_p(1)$ and calculates with the aid of Eq. (41) a value $[\Delta \vec{V}_p]$. The next iteration step assumes

$$[\vec{V}_p(2)]_2 = \vec{V}_p(1) + [\Delta \vec{V}_p]_1 \quad (44)$$

and so forth (the subscript number denotes the number of iteration step). The iteration is continued until the angle between the vectors $[(\vec{V}_p)_{av}]_{n+1}$ and $[(\vec{V}_p)_{av}]_n$ is less than 0.001 radian. The resulting values for $\vec{V}_p(2)$ and $P_2(x_2, y_2)$ provide the starting point for a similar iteration for the next field.

The iteration operation is illustrated in Fig. 8. The jet and its Mach net are shown schematically. The bold line $AP_1P_2P_3P_4P_5A$ is the final result, i.e., the particle trajectory in the jet which is composed of many tiny chords in successive flow fields. The dashed segments are the tentative pathways of the particle during the iteration process. The subscript number of the parenthesis denote the number of the iteration steps. The calculation is continued to the point where the air pressure has decreased to 1% of the pressure of the nozzle exit plane. Beyond this point, the interaction between air flow and particles is considered negligible and the particles are assumed to travel on

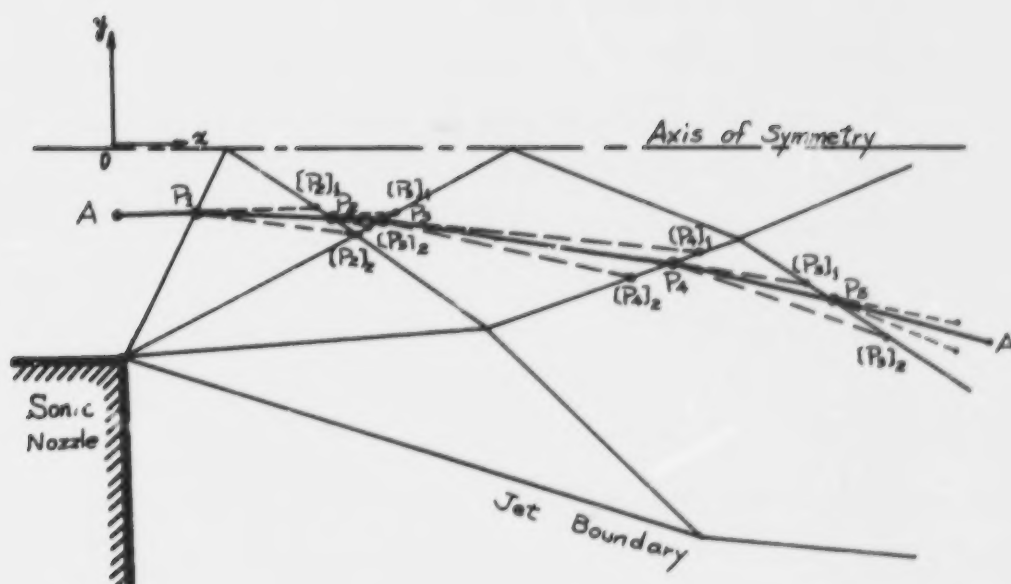


Fig. 8 Particle Trajectory Calculation by Iteration Method

rectilinear paths.

The calculations are started at the nozzle exit which requires the knowledge of the particle location $P(o, y)$ and velocity $\vec{V}_p(o, y)$ in the nozzle exit plane. These initial conditions can be determined by either calculating the particle trajectories inside the nozzle or by using nozzles which generate a particle flow with known properties of the nozzle exit.

The computations of the particle trajectories were carried out with a Univac 1108 computer. The computer program is given with a sample calculation in Appendix C.

III. DESIGN OF NOZZLE A

The design of a nozzle which generates a flow of particles which are in velocity equilibrium with the air flow inside the nozzle is described in the following section. The particles cross the nozzle exit plane perpendicularly and have the same velocity as the air flow. Thus the particle velocity is known at the nozzle exit.

Particles remain in velocity equilibrium with the gas flow, if the particles undergo the same acceleration as the fluid element in which they are suspended. Let us consider two cross sections in the nozzle which are a distance Δx apart, and assume that the particles are in velocity equilibrium with the gas at the upstream cross-section, A. The acceleration of the particle between these sections is given by Eq. (41), where $\vec{v}_g$ and $\vec{v}_p$ can be approximated by:

$$\begin{aligned}\vec{v}_g &= \vec{v}_g(A) + \left[\frac{d\vec{v}_g(A)}{dt} \right] \cdot \Delta t \\ \vec{v}_p &= \vec{v}_p(A) + \left[\frac{d\vec{v}_p(A)}{dt} \right] \cdot \Delta t\end{aligned}\tag{45}$$

Equation (45) together with $\vec{v}_p(A) = \vec{v}_g(A)$ introduced into Eq. (41) gives:

$$\frac{\frac{d\vec{v}_p(A)}{dt}}{\frac{d\vec{v}_g(A)}{dt}} = \frac{\frac{\Delta t}{C\tau}}{1 + \frac{\Delta t}{2C\tau}}\tag{46}$$

which means that the particles remain in equilibrium with the air flow,

$$\text{if } \frac{\Delta t}{2C\tau} \gg 1.$$

The shape of the nozzle wall was determined by dividing the nozzle into small discs with the thickness

$$\Delta x = \left[v_{gx}(A) + \frac{\Delta v_{gx}}{2} \right] \cdot \Delta t \quad (47)$$

and by calculating the disc radius R_N under the assumption that the problem may be approximated by one-dimensional, frictionless subsonic flow from (Shapiro, loc. cit., Vol. 1, p. 86):

$$R_N = R_N^* \frac{1}{\sqrt{M}} \cdot \left\{ \left(\frac{2}{k+1} \right) \cdot \left[1 + \left(\frac{k-1}{2} \cdot M^2 \right) \right] \right\}^{\frac{k+1}{4(k-1)}} \quad (48)$$

The numerical calculations were carried out with: $\Delta t = 0.31 \cdot 10^{-5}$ sec., which corresponds to a $C\tau$ value of a latex sphere with $d = 0.714 \mu\text{m}$ at $p = 100\text{mmHg}$ and $T = 298^\circ\text{K}$; $\Delta v_{gx} = 500 \text{ cm/sec}$; $k = 1.4$; $M = \frac{v_{gx}}{v_g^*}$ with $v_g^* = 34600 \text{ cm/sec}$; and a nozzle exit radius $R_N^* = 250 \mu\text{m}$.

The result of the calculation is given in Table 3. The calculated nozzle shape is shown in Fig. 9 together with the contour of the actually fabricated nozzle. The actual nozzle A was shortened to simplify fabrication and to avoid the possible occurrence of choking flow inside the nozzle due to friction. The actual nozzle has an exit radius $R_N^* = 238 \mu\text{m}$.

Axial parallel flow of the particles at the exit is guaranteed by the $750 \mu\text{m}$ long section of constant area duct at the nozzle end. The shortening of the nozzle, however, can result in a certain concentration

Table 3

Calculation of Nozzle Contour (Nozzle A) with Design Parameters

$$\begin{aligned}
 P_0 &= 100 \text{ mmHg} & R_N^* &= 250 \mu\text{m} \\
 d_0 &= 0.7 \mu\text{m} & C_T &= 8.06 \cdot 10^{-6} \text{ sec} \\
 T_0 &= 2980 \text{ K} & V_j^* &= 34600 \text{ cm/sec}
 \end{aligned}$$

No. of Sections	V_{px} (cm/sec)	M	R_N/R_N^*	$\bar{V}_{px}$ (cm/sec)	Δx (μm)	$x = \sum \Delta x$ (μm)	R_N (μm)
1	500	0.0145	6.33	250	8	8	1585
2	1000	0.029	4.47	750	23	31	1115
3	1500	0.0433	3.66	1250	38	69	915
4	2000	0.0578	3.16	1750	53	122	790
5	2500	0.0723	2.83	2250	69	191	713
6	3000	0.087	2.54	2750	84	275	635
7	3500	0.1	2.42	3250	100	375	605
8	4000	0.11	2.3	3750	111	486	575
9	4500	0.13	2.12	4250	130	616	530
10	5000	0.145	2.01	4750	145	761	503
11	5500	0.159	1.92	5250	161	922	480
12	6000	0.165	1.84	5750	176	1098	460
13	6500	0.188	1.78	6250	191	1289	445
14	7000	0.203	1.71	6750	206	1495	428
15	7500	0.217	1.66	7250	222	1717	415
16	8000	0.231	1.61	7750	237	1954	403
17	8500	0.246	1.56	8250	252	2206	390
18	9000	0.260	1.53	8750	267	2473	383
19	9500	0.275	1.49	9250	283	2756	373
20	10000	0.289	1.46	9750	298	3054	365
21	10500	0.304	1.43	10250	314	3368	358

Table 3 (Cont.)

No. of Sections	V_{px} (cm/sec)	M	R_N/R_N^*	$\bar{V}_{px}$ (cm/sec)	Δx (μm)	$x = \sum \Delta x$ (μm)	R_N (μm)
22	11000	0.318	1.4	10750	329	3697	350
23	11500	0.333	1.36	11250	344	4041	340
24	12000	0.347	1.34	11750	360	4401	335
25	12500	0.361	1.32	12250	375	4776	330
26	13000	0.376	1.3	12750	390	5166	325
27	13500	0.39	1.28	13250	405	5571	320
28	14000	0.404	1.26	13750	421	5992	315
29	14500	0.420	1.24	14250	436	6428	310
30	15000	0.433	1.23	14750	452	6880	308
31	15500	0.448	1.21	15250	467	7347	303
32	16000	0.462	1.2	15750	482	7829	300
33	16500	0.477	1.18	16250	498	8327	295
34	17000	0.491	1.17	16750	513	8840	293
35	17500	0.507	1.15	17250	528	9368	288
36	18000	0.52	1.145	17750	543	9911	286
37	18500	0.535	1.135	18250	558	10469	284
38	19000	0.55	1.12	18750	574	11043	280
39	19500	0.563	1.11	19250	590	11633	—
40	20000	0.578	1.105	19750	605	12238	—
41	20500	0.593	1.1	20250	620	12858	—
42	21000	0.607	1.09	20750	635	13493	—
43	21500	0.622	1.08	21250	650	14143	≈ 250

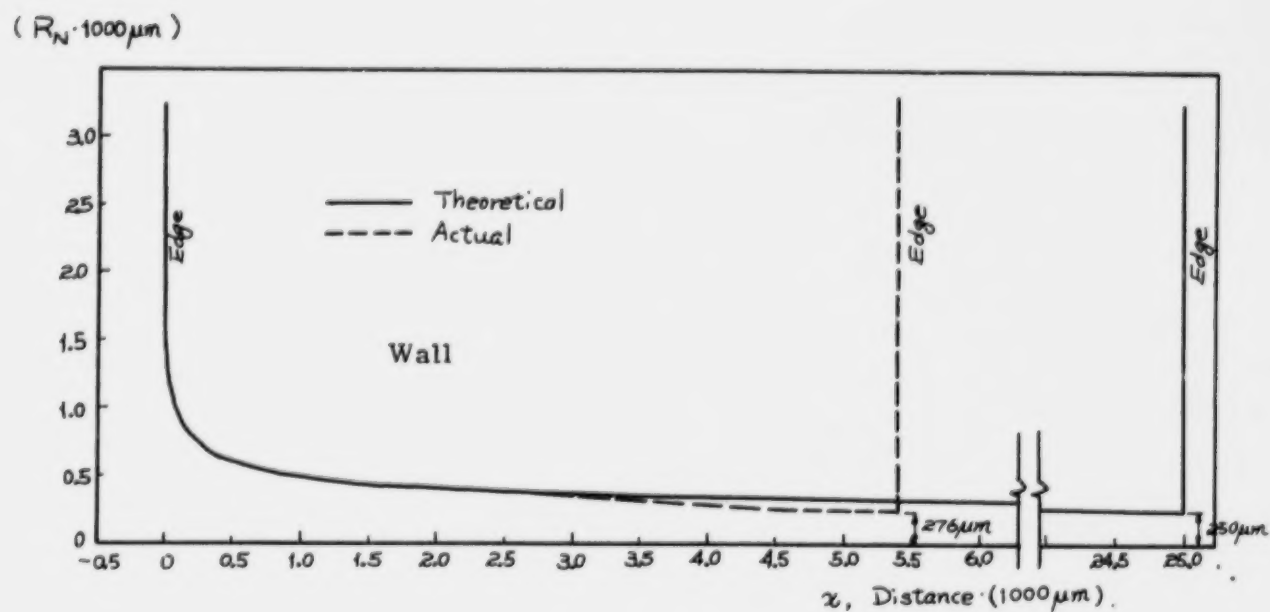


Fig. 9 Nozzle Contour

x = distance from nozzle inlet
 R_N = radius of nozzle cross section

of the particle flow near the nozzle axis, if the particles enter the constant-area section with a velocity component perpendicular to the axis. Inasmuch as $C\tau$ decreases with increasing pressure and decreasing particle size, the particles will follow the air streamlines closer the smaller they are and the larger the source pressure is.

IV. EXPERIMENTAL PROCEDURE

The aerosol beam generator is shown schematically in Fig. 10. Compressed air, filtered by a millipore filter F with average pore size of $0.22\ \mu\text{m}$, operates the Dautrebande type nebulizer, A, which contains an aqueous suspension of polystyrene latex spheres. The data of the test aerosol, as provided by the Dow Chemical Company, are given in Table 4. The density of the aerosol is $1.05\ \text{g/cc}$. The particle weight fraction in the nozzle flow was always less than $10^{-5}(\text{g/g})$.

Water vapor is removed in the cold trap C, which is cooled with a dry ice-methanol mixture. The hold up vessel W allows the mixture to retain room temperature.

The atomization process in the nebulizer causes electrical charging of the particles. This electrical charge is greatly reduced by passing the aerosol through an enclosed Kr-85 source, I (strength 1 mC). The residence time in the krypton source is about 25 seconds. The air-particle mixture is drawn through a capillary K by the pump, P, (Welsh, 1400B, $0.35\ \text{l/sec}$) into the source chamber, S, the beam producing system. The static pressure measured by manometer G_1 is called source pressure, P_o . The source pressure can be varied from 760 mmHg down to 15 mmHg, and is controlled either by the size of the capillary K and/or by the opening of the needle valve, V, leading to pump P_1 . The glass capillaries K used to control the source pressure P_o are about 10 cm in length with inner diameters ranging from $75\ \mu\text{m}$ to $500\ \mu\text{m}$.

The gas-particle mixture expands through nozzle N into the measuring chamber E, which is continuously evacuated by the pump P_2 (Welch 1397; $7.17/\text{sec.}$) in order to keep the background pressure, P_e

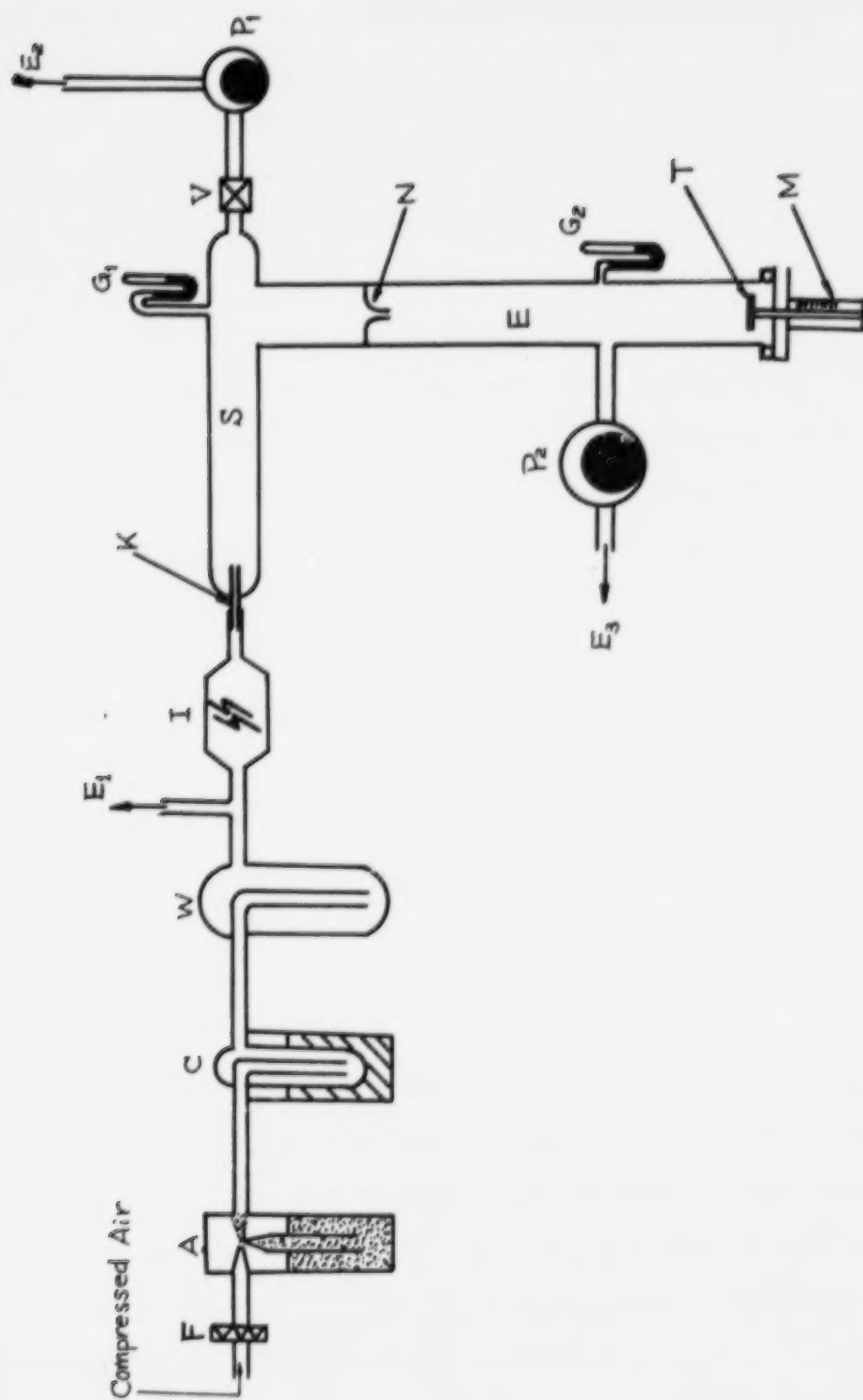


Fig. 10 Schematic Diagram of the Particle Beam Apparatus

Table 4

The Latex Particle Diameter Used in this Investigation, and
its Batch Number, Standard Deviation, and Percentage
Content of Solid

Batch No.	Latex particle diameter, d_p (μm)	Standard Deviation, (μm)	% of solid (g/g)
LS-464-E	1.305	0.0156	1:300
LS-1138-B	1.011	0.0054	1:300
LS-1012-E	0.714	0.0053	1:300
LS-1029-E	0.5	0.0027	1:300
LS-061-A	0.365	0.0079	1:100
LS-1047-E	0.234	0.0026	1:1000
LS-052-A	0.126	0.0043	1:1000

(the static pressure in the measuring chamber E), below 1.5 mmHg. G_1 and G_2 are pressure gages. The accuracy of G_1 is about 1 mmHg, and 0.1 mmHg for G_2 . E_1 , E_2 , and E_3 designate exhausts for the gas-particle mixture.

The gas stream, which is accelerated to sonic velocity in the nozzle, N, acts as a propellant for the particles, thus generating a beam of directed particles with particle velocities comparable to the velocity of the emerging gas-jet. The particles in the beam are collected on a microscope cover-glass target, T, which is mounted perpendicular to the beam axis with the aid of a target holder. A thin coating of silicone oil (Dow Corning 200, Viscosity 3000 poises) is applied on the target to minimize the slipping and reflection of the particles from the target surface. The distance between nozzle exit plane and target surface (called target distance, x_t) is adjustable between 0.5 and 130 mm with the micrometer M with an accuracy of about 0.05 mm. The aerosol deposit was investigated and photographed in the dark field of an optical microscope.

The beam generator is shown in scale in Fig. 11. The nozzles are bored into brass discs of 2.5 cm diameter. They are easily exchanged in the nozzle holder which is flanged between source and measuring chamber. Rubber o-rings serve as packing.

Experiments were conducted with 3 different nozzles besides the one described in the previous chapter. Their main characteristics are:

Nozzle A: (see Fig. 9)

Nozzle B: (capillary)

A capillary duct with almost constant cross-section area throughout the whole nozzle length and 90° sharp-edged inlet

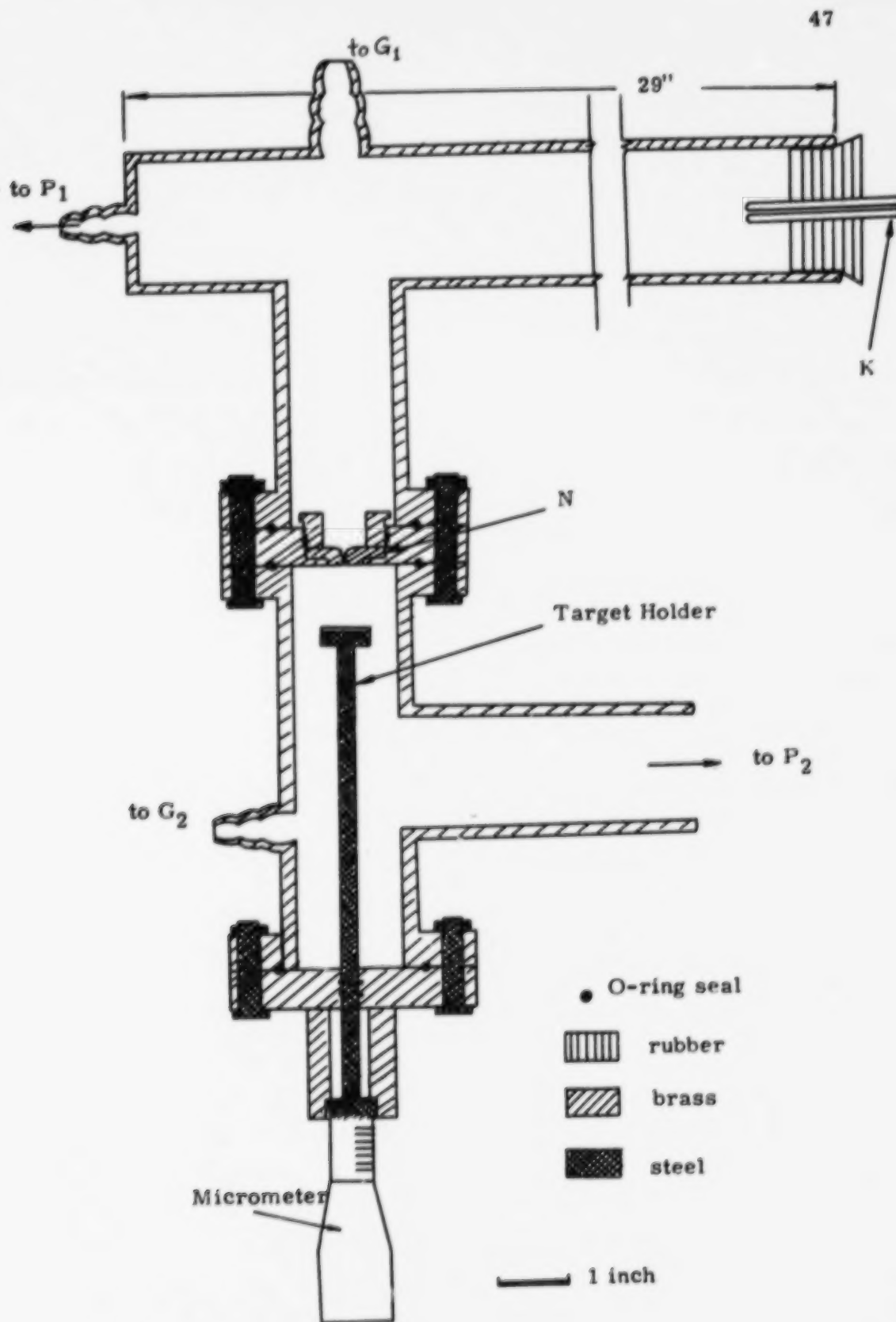


Fig. 11 The Beam Generator

and outlet. Limited by the fabricating technique, the diameter of the inlet is about 15% larger than the outlet.

Nozzle C:

A convergent nozzle with a constant cone angle of $2\alpha_N = 10.9^\circ$ and circular cross-section. The edges of the nozzle are sharp and perpendicular to the axis of the nozzle.

The nozzle contours are shown in Fig. 12 and their geometrical data are summarized in Table 5.

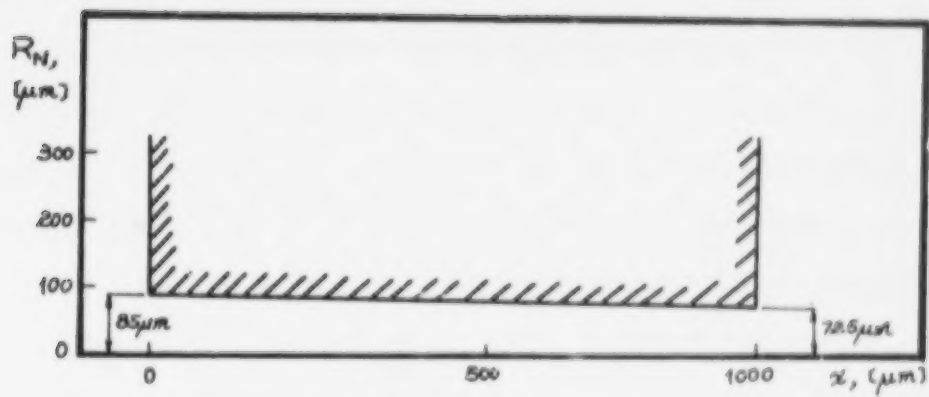
The pressure ratio P_e/P_o is always kept smaller than the critical pressure ratio which is 0.528 for air. Therefore, the gas reaches sonic velocity at the throat and the volume flow rate is given under assumption of frictionless and adiabatic flow by:

$$Q = \frac{A^* P_o}{\rho_g} \sqrt{\frac{k}{R_g T_o} \cdot \left[\frac{2}{k+1} \right]^{\frac{k+1}{k-1}}} \quad (49)$$

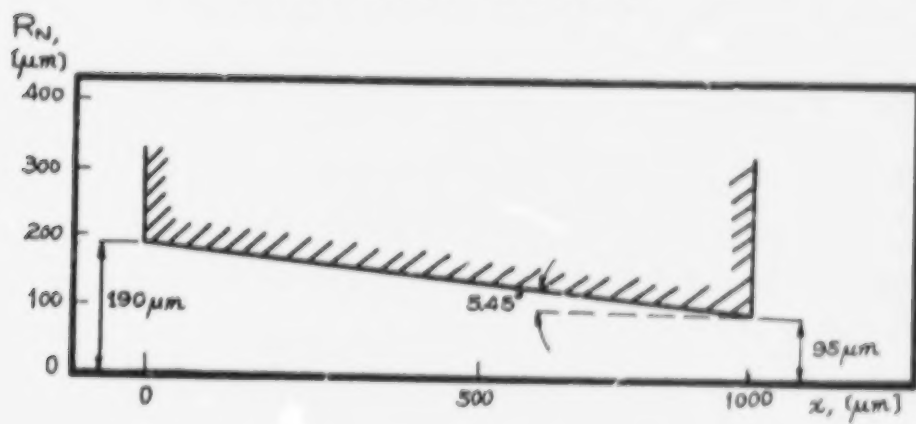
with R_g = gas constant

A^* = area of nozzle throat

Fig. 13 shows a comparison between the flow rate calculated by Eq. (49) and measured flow rates for different nozzles. The agreement is good for nozzle A, i. e., the nozzle with a large cross-section. The experimental flow rates are somewhat smaller than the calculated ones for nozzles B and C. This reduction in the flow rate is possibly due to friction occurring on the nozzle wall.



(a) The Contour of Nozzle B



(b) The Contour of Nozzle C

Fig. 12

Table 5

Brief Description of Nozzle Contours

Nozzle	Description	length	cone angle at the outlet $2\alpha_N$	inlet diameter (μm)	outlet diameter (μm)
A	Nozzle with transitional inlet & cap- illary outlet	54,000 μm	0°	—	576
B	capillary	1000 μm	0°	170	145
C	Convergent nozzle with cone angle $2\alpha_N = 10.9^\circ$	1000 μm	10.9°	380	190

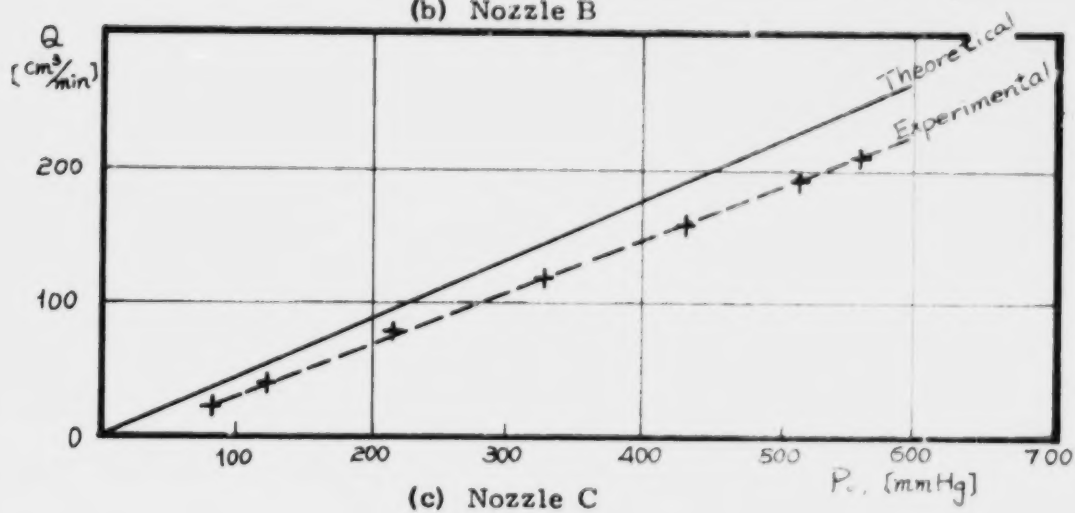
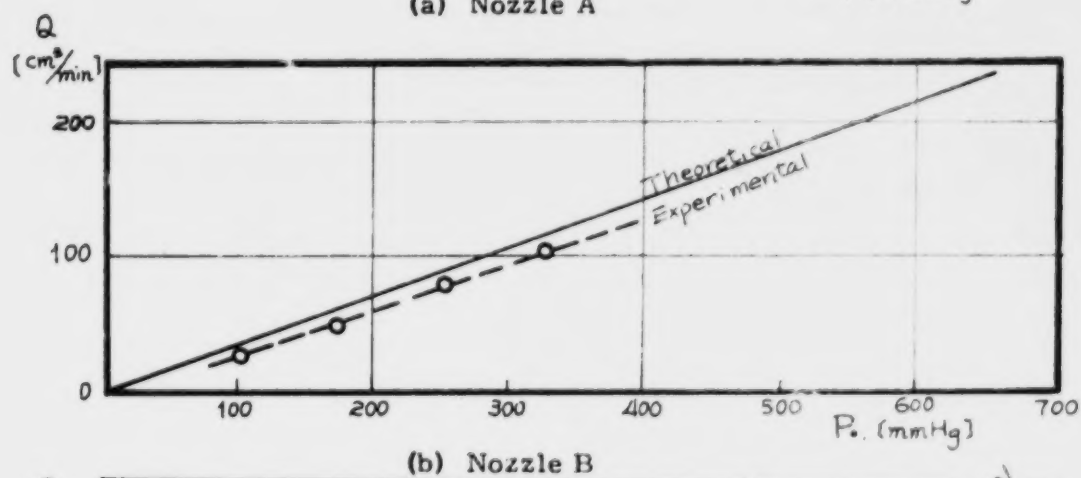
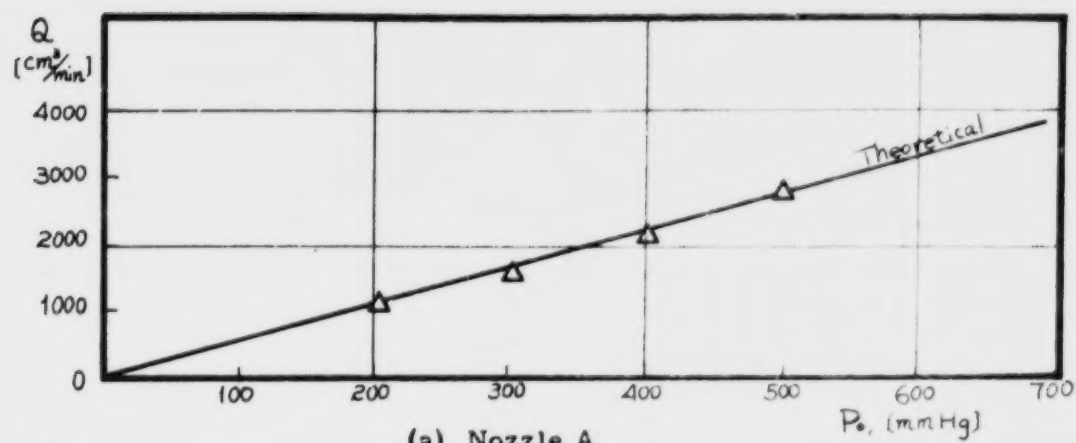


Fig. 13 Flow Rate Dependence Upon the Source Pressure

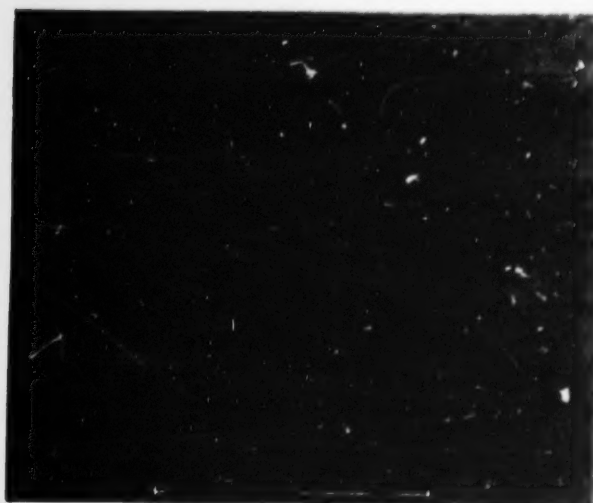
V. EXPERIMENTAL RESULTS AND DISCUSSION

Nozzle A

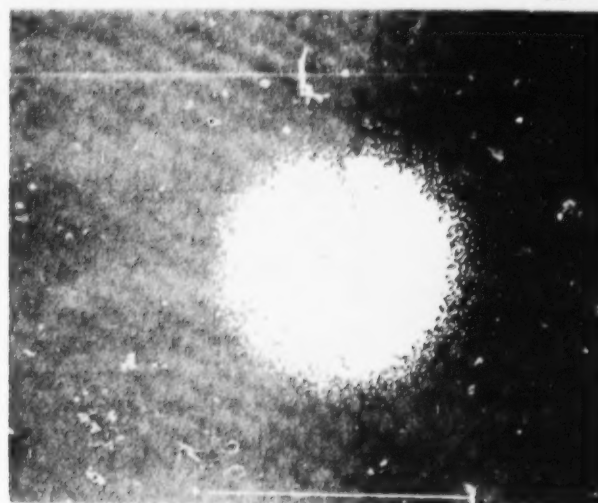
Figure 14a shows a typical picture of the particle deposit on the collector. The boundary of the deposit is well defined and the particle density is rather uniform across the deposit disc. The ability of the instrument to discriminate between the size of various particles contained in an aerosol mixture is demonstrated in Figs. 14b and 14c. These figures show the deposit resulting from a mixture of two and three different particle sizes. The boundary of the deposit disc of each particle size is clearly discernible, and the diameter of each disc, which is directly related to the particle size, can be easily measured microscopically.

A major objective of these investigations was to test the applicability of the theory outlined in Chapter II by comparing computed particle trajectories in the free jet with experimental data.

The particles emerge from Nozzle A as a beam with circular cross section of radius r_j . Particles traveling through the periphery of this cross section form the edge of the deposit on the target which is located at the distance x_t downstream from the nozzle. This fact allows to compare the computed trajectories for these particles with the experiment. The procedure is illustrated by Fig. 15. The solid lines are calculated trajectories for particles traversing the nozzle exit plane at different distances r_j from the axis. The open circles show experimentally determined radii of particle deposits collected at

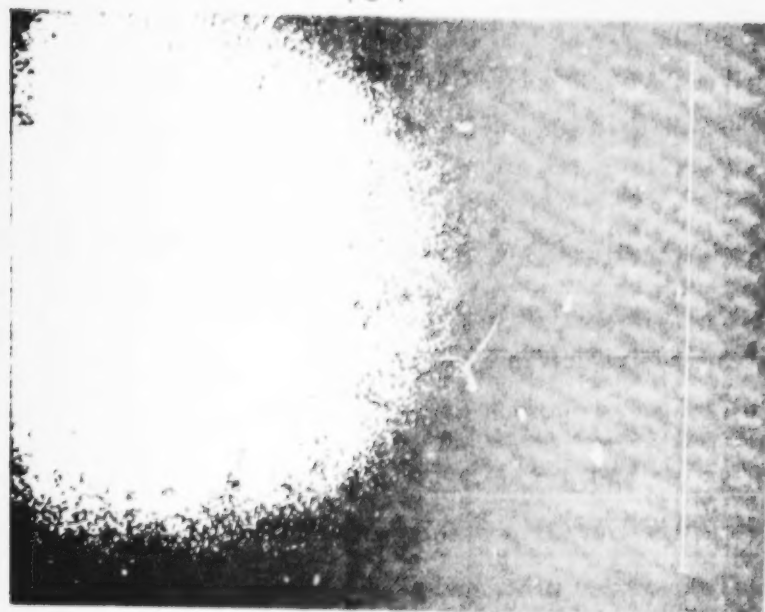


(a)



(b)

(c)



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Fig. 14: Examples of particle deposit. Indicated scale corresponds to $500\mu\text{m}$.

- (a) Particle diameter $d_p = 0.714\mu\text{m}$; source pressure $P_o = 250\text{mmHg}$; $x_t = 3.0\text{mm}$.
- (b) Particle mixture with $d_p = 1.3\mu\text{m}$ and $d_p = 0.714\mu\text{m}$; $P_o = 250\text{mmHg}$; $x_t = 2.5\text{mm}$.
- (c) Particle mixture with $d_p = 1.3\mu\text{m}$; $d_p = 0.714\mu\text{m}$; $d_p = 0.365\mu\text{m}$; $P_o = 250\text{mmHg}$; $x_t = 2.0\text{mm}$.

various distances from the nozzle. Satisfactory agreement between theory and experiment is indicated if the data points follow closely one of the calculated trajectories, as it is, e.g., demonstrated by Fig. 15.

Fig. 16 summarizes this comparison between computed and measured particle trajectories for various particle sizes in dependence upon the source pressure. The data points correspond to measured D_{dep} values (diameter of the deposit), whereas the solid lines refer to the corresponding beam diameter in the nozzle exit plane, d_j in (μm), upon which the calculation was based. The graphs show a good agreement between theory and experimental particle trajectories for all particle sizes and source pressures larger than 100 mmHg, i.e., within the design parameters of the nozzle. Thus the theory describes the particle beam satisfactory and provides a basis for the design of particle beams with special properties.

The computation of the particle trajectories was based upon Stokes drag and two dimensional flow. The former is applicable only for small Reynolds numbers and the latter describes the flow from an infinite slit rather than from a concentric nozzle. These simplifying assumptions, however, are justified for an approximation of the experimental situation.

The particle Reynold number in the free jet region is given by:

$$\text{Re} = \frac{\rho_p d_p |\vec{v}_g - \vec{v}_p|}{\eta_g} \quad (50)$$

For experiments with source pressure larger than 100 mmHg, Re reaches a maximum value of $\text{Re} = 5$ for particles with $d_p = 1.3 \mu\text{m}$ at $P_0 = 100 \text{ mmHg}$. The components of the Reynolds number parallel and perpendicular to the beam axis are the same within 15%. This leads to the conclusion that Stokes formula should represent the particle drag within about 10%.

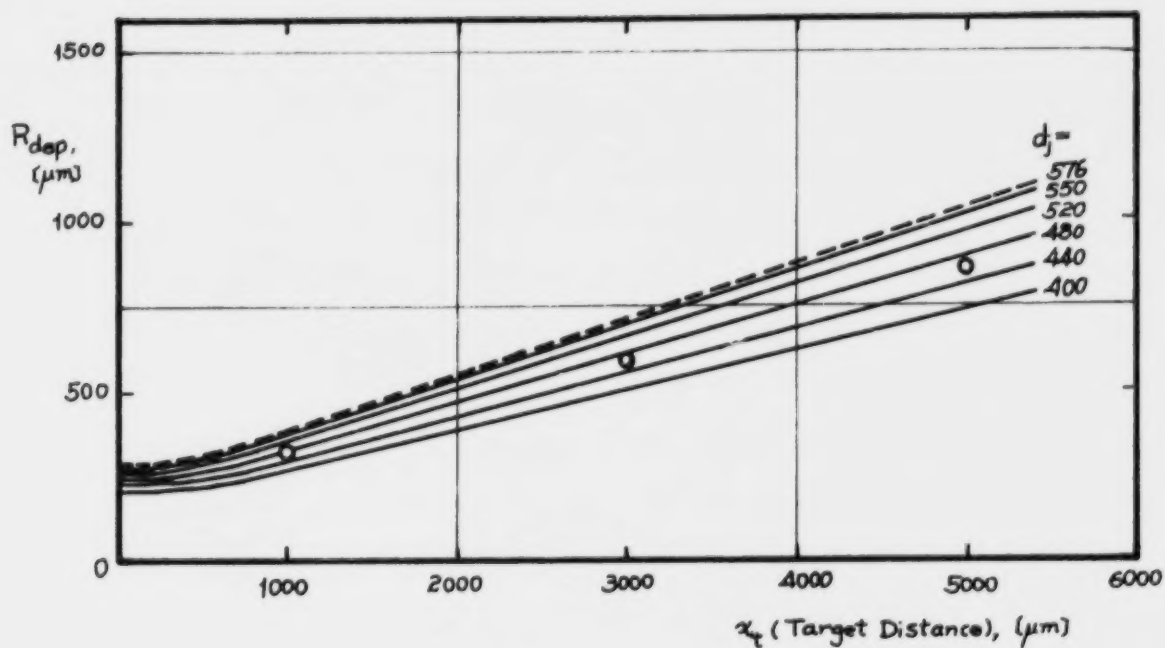


Fig. 15 Theoretical Trajectories and Experimental Values for Nozzle A. $P_o = 250 \text{ mmHg}$; $P_e = 1 \text{ mmHg}$; $d_p = 0.5 \mu m$; $D^* = 576 \mu m$

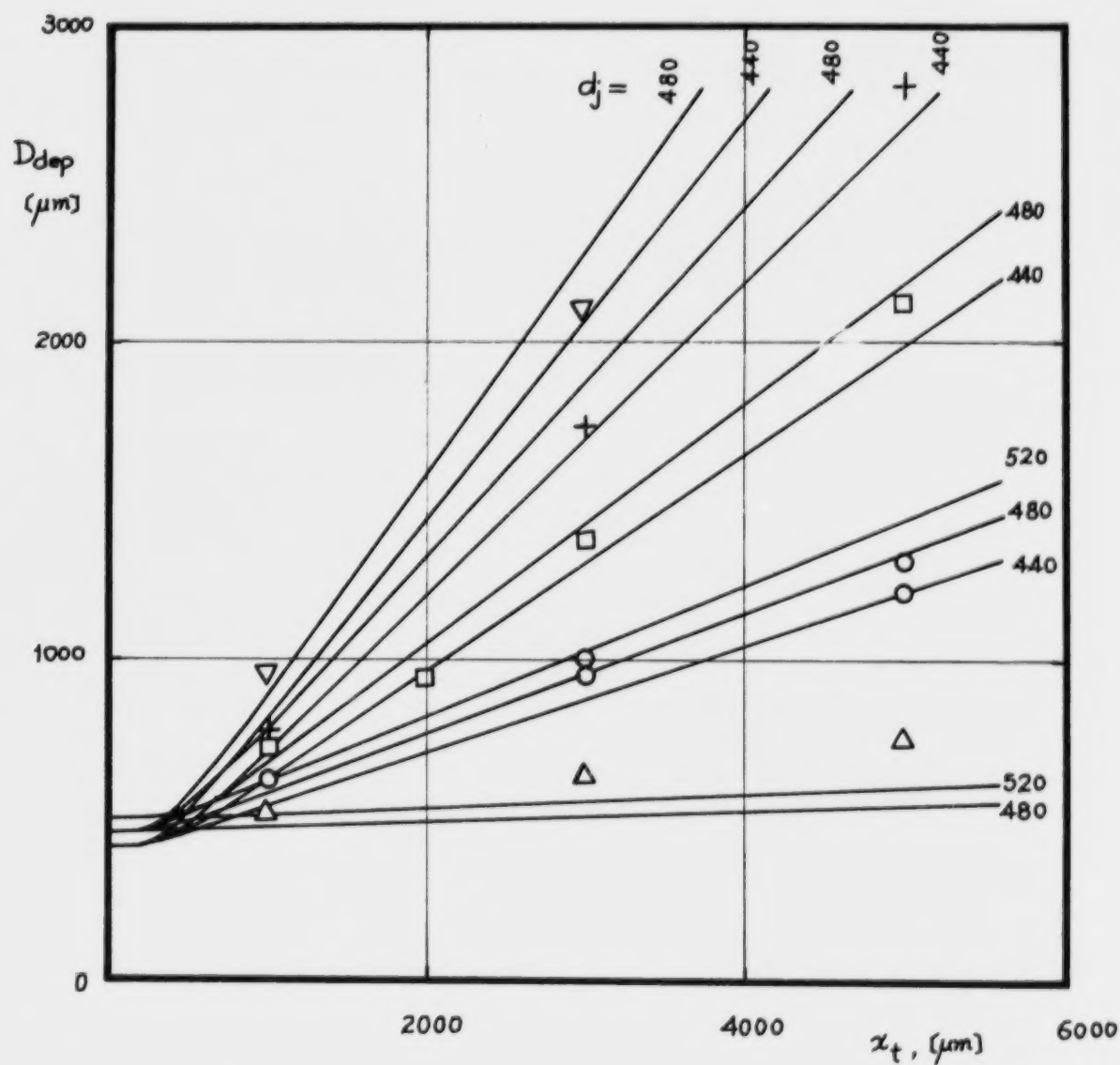


Fig. 16(a) Comparison Between Calculated and Measured Particle Deposit Diameters for Different Particle Sizes (Nozzle A)

(a) $d_p = 0.365 \mu m$

∇ : $P_o = 760$ mm Hg
 $+$: $P_o = 450$ mm Hg
 $\square$: $P_o = 250$ mm Hg
 $\circ$: $P_o = 100$ mm Hg
 $\triangle$: $P_o = 30$ mm Hg

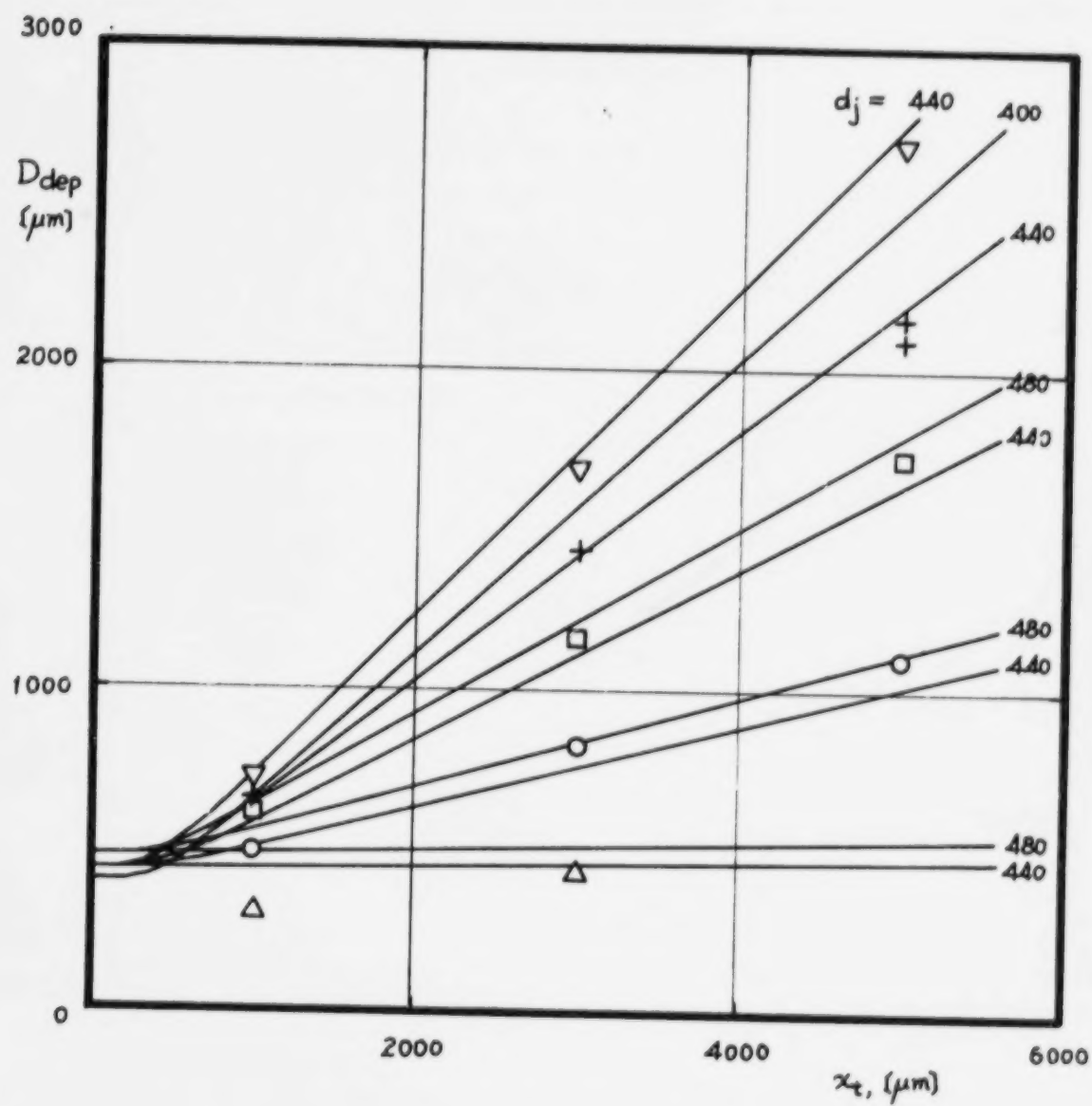


Fig. 16(b) $d_p = 0.5 \mu\text{m}$

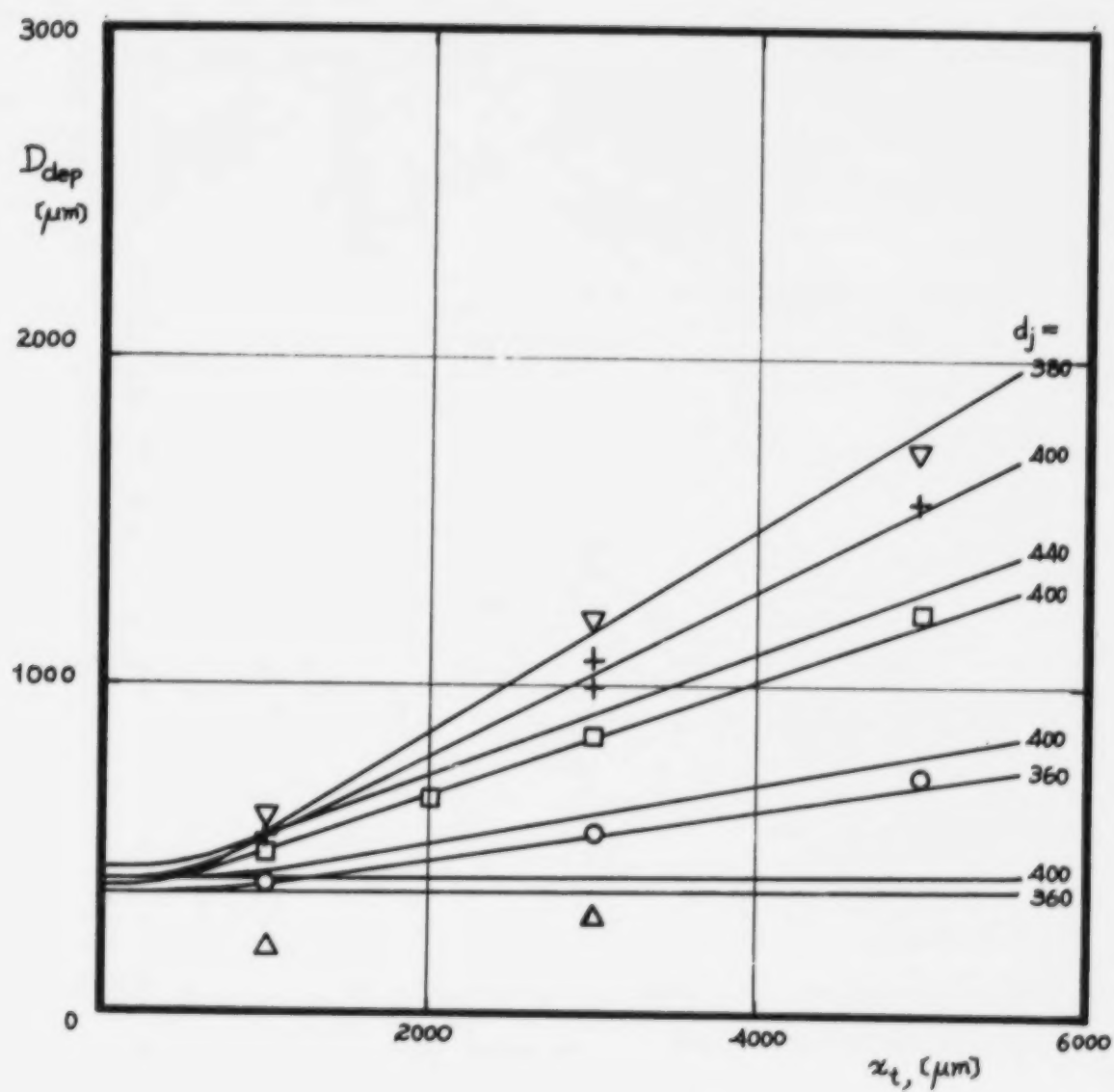


Fig. 16(c) $d_p = 0.714 \mu m$

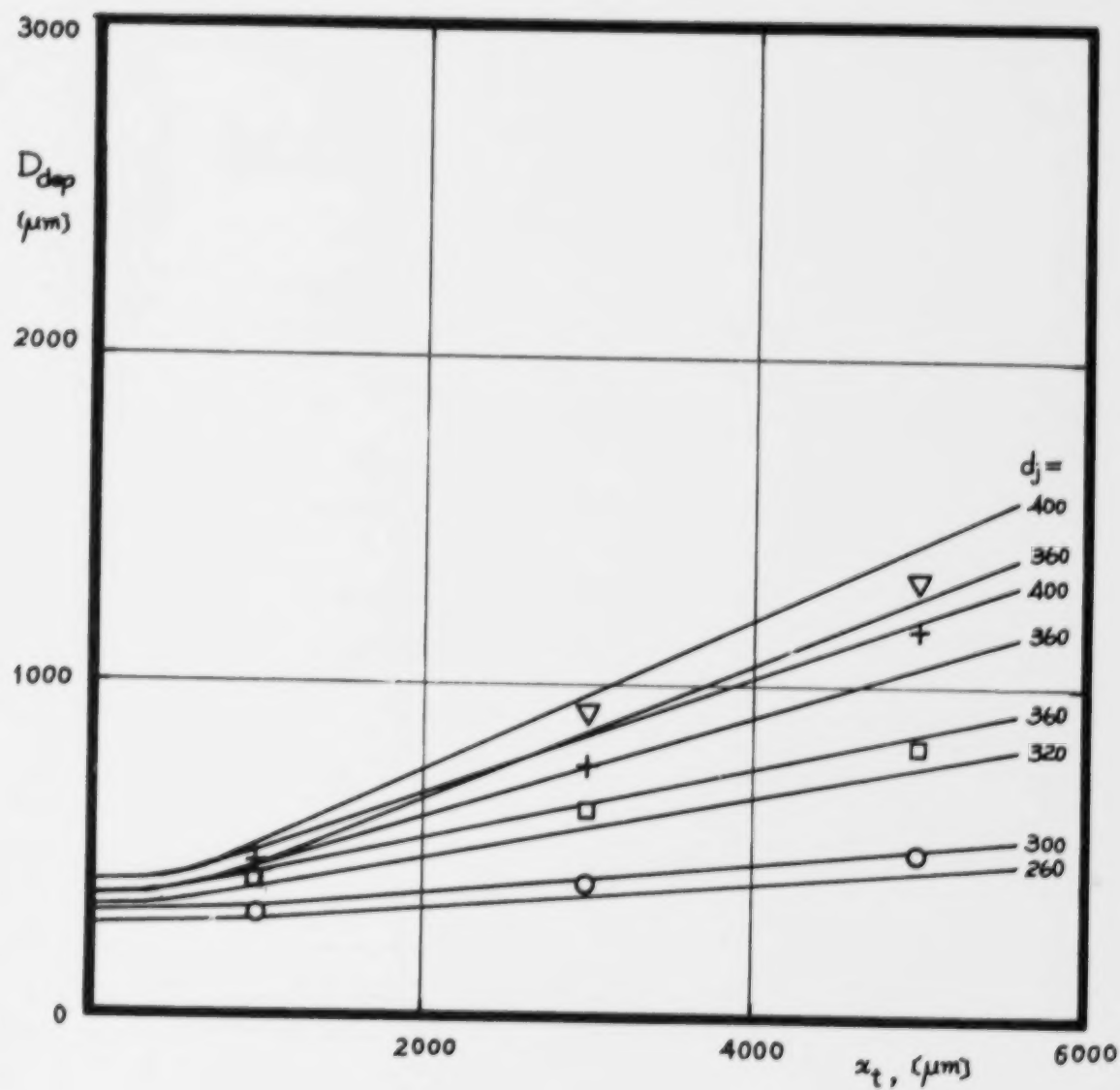


Fig. 16(d) $d_p = 1.011 \mu m$

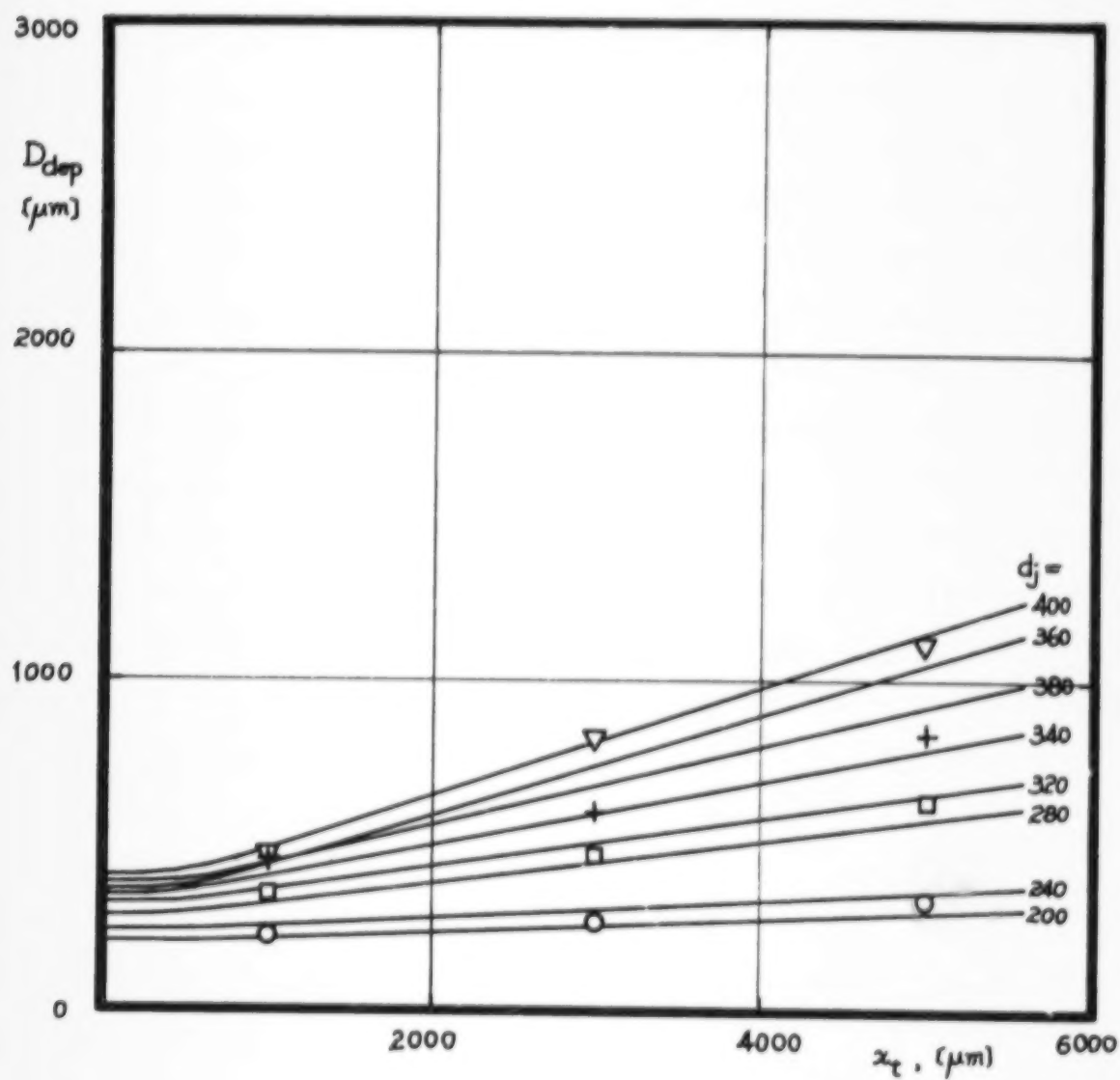


Fig. 16(e) $d_p = 1.3 \mu m$

The jet expanding from an axis symmetrical nozzle is three-dimensional, whereas the theoretical calculations were based upon two-dimensional flow. However, the direction of the characteristics in the two-dimensional and in the rotationally symmetrical problem should be the same in the limit at the nozzle exit. Therefore we can expect that the three-dimensional problem can be approximated by the two-dimensional in the vicinity of the nozzle throat, i. e., the area of main interest for the interaction between particles and gas jet.

The calculations indicate and the measurements verify (Fig. 16) that the interaction between the air jet and the particle beam become negligible at about two nozzle diameters downstream from the nozzle due to the rarefaction of the air jet. If, therefore, the target is mounted at a distance larger than $2D^*$ downstream from the nozzle, all particles irrespective of their size are impinged on the collector. This is in contrast to the well known impactor principle, where particles below a given size are carried around the collector with the air stream.

The experiments show that the nozzle exit ranges from about $d_j = 480 \mu\text{m}$ for $0.234 \mu\text{m}$ particles and $p_o = 760 \text{ mmHg}$ down to about $d_j = 300 \mu\text{m}$ for $1.3 \mu\text{m}$ particles and $p_o = 250 \text{ mmHg}$. Thus d_j is smaller than the throat diameter D^* . This constriction of the beam is caused, at least in part, by the shortening of the nozzle, as previously discussed.

Fig. 17 shows the experimentally determined relationship between the diameter of the deposit disc, D_{dep} , and the particle size for various source pressures. The measured values were connected by solid lines, which can be considered calibration curves for the instrument. The results indicate that the method allows the measurement of the particle

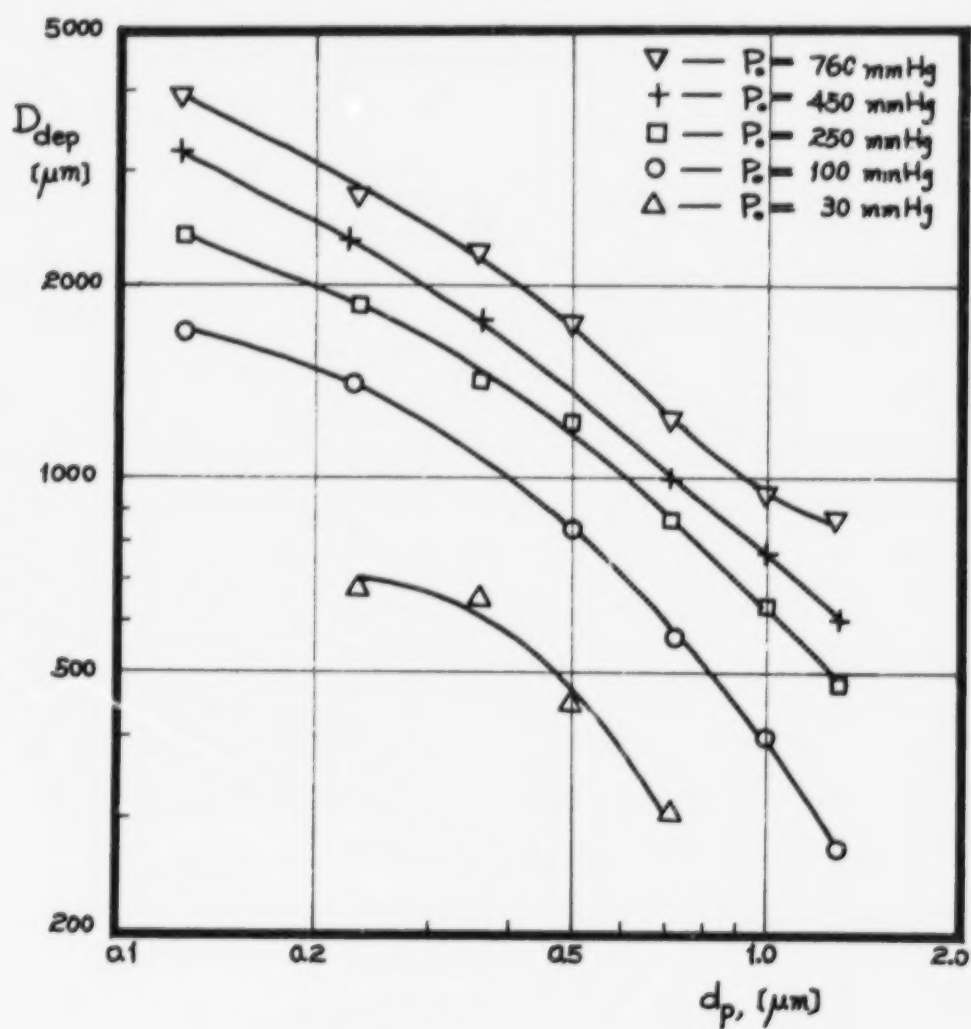


Fig. 17 Dependence of deposit diameter upon particle size for spherical particles with the density $\rho_p = 1.05 \text{ g/cm}^3$; $x_t = 3.0 \text{ mm}$

size of monodisperse spheres in the tested range of $0.126 \mu\text{m} \leq d \leq 1.3 \mu\text{m}$ with an accuracy of about 5%. It is very likely that this range can be extended toward smaller as well as larger particle sizes.

The particle density distribution across the deposit disc was determined by photographing the deposit and subsequent counting the particle number per unit area. Fig. 18 demonstrates the procedure. Several traverses, originating in the center, are drawn on the photograph. A ruler, subdivided into small squares, is placed upon a marked traverse and the particle density, I_i , at a given distance, r_i , from the center is determined by counting the particles in the square (unit area).

Figures 19 and 20 show the result of the deposit density evaluation. The distance from the deposit center is plotted in terms of r_i/R_{dep} and the particle density, I_i , is expressed in percent of the density at the distance $r_i = 0.2 R_{\text{dep}}$. This facilitates the comparison of measurements for different particle sizes and source pressures. The error bars indicate the standard deviation of the particle density. Fig. 19 indicates a uniform deposit for particles larger than $0.5 \mu\text{m}$. For smaller particles the density seems to decrease slowly with increasing distance from the center. Fig. 20 gives several examples for different particle sizes and source pressures, which show the same behavior.

Knowing the particle density distribution across the deposit and the relation between deposit disc diameter and particle size, one can determine the size distribution of unknown aerosols. This is demonstrated for a sample of atmospheric aerosol in Fig. 21.

The cumulative size distribution was determined by evaluating the particle density across the deposit assuming the applicability of calibration

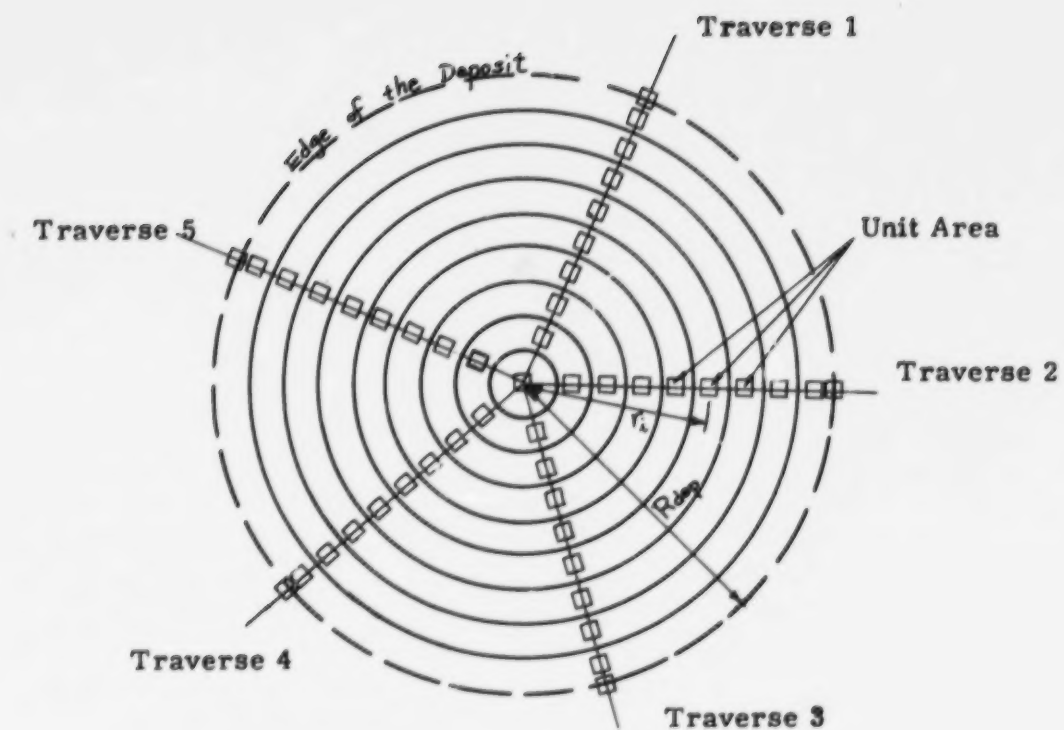


Fig. 18 Particle Counting Technique

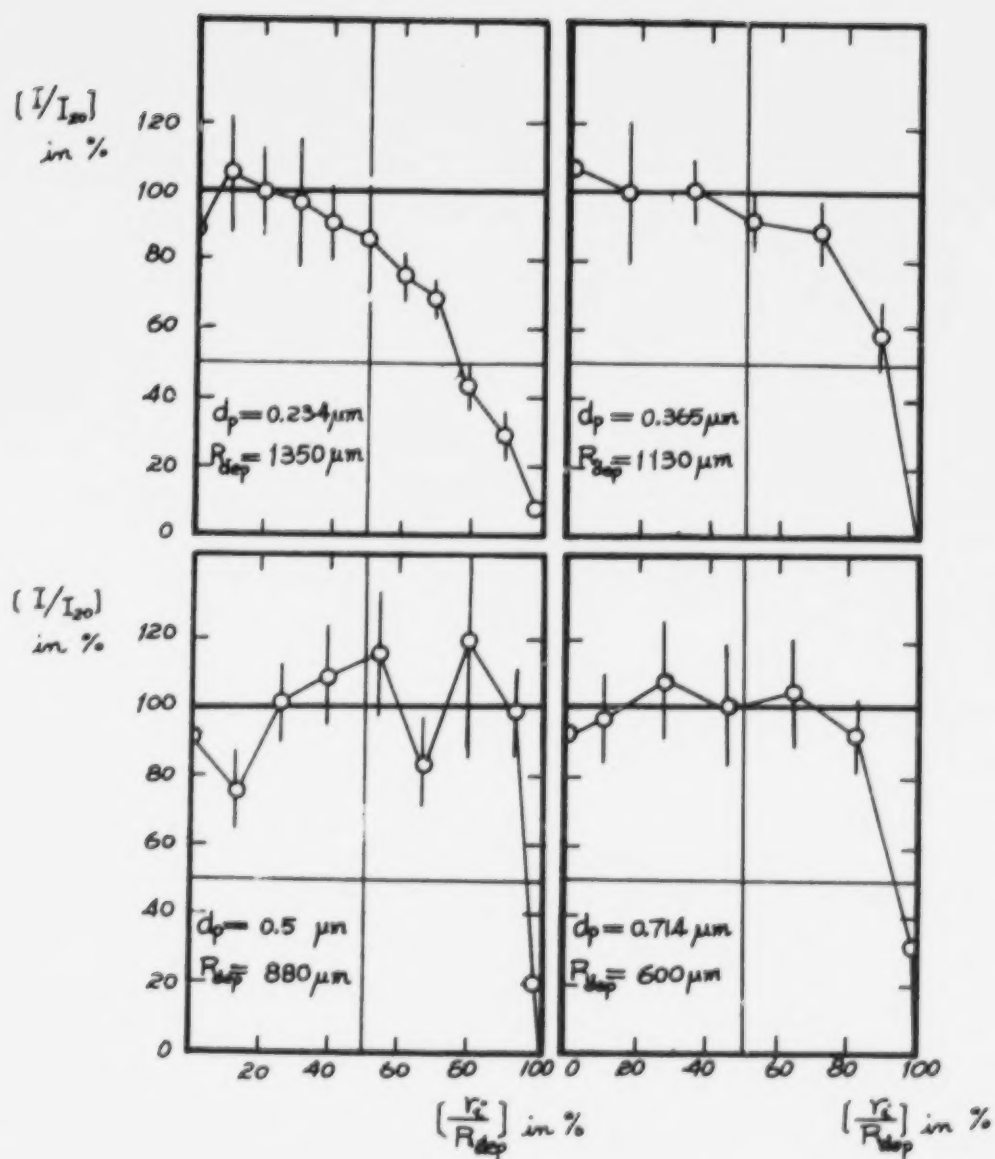


Fig. 19 Particle Density Distribution across the Deposit. $p_0 = 760 \text{ mmHg}$; $x_t = 3 \text{ mm}$

(error bar represents standard deviation of the data)

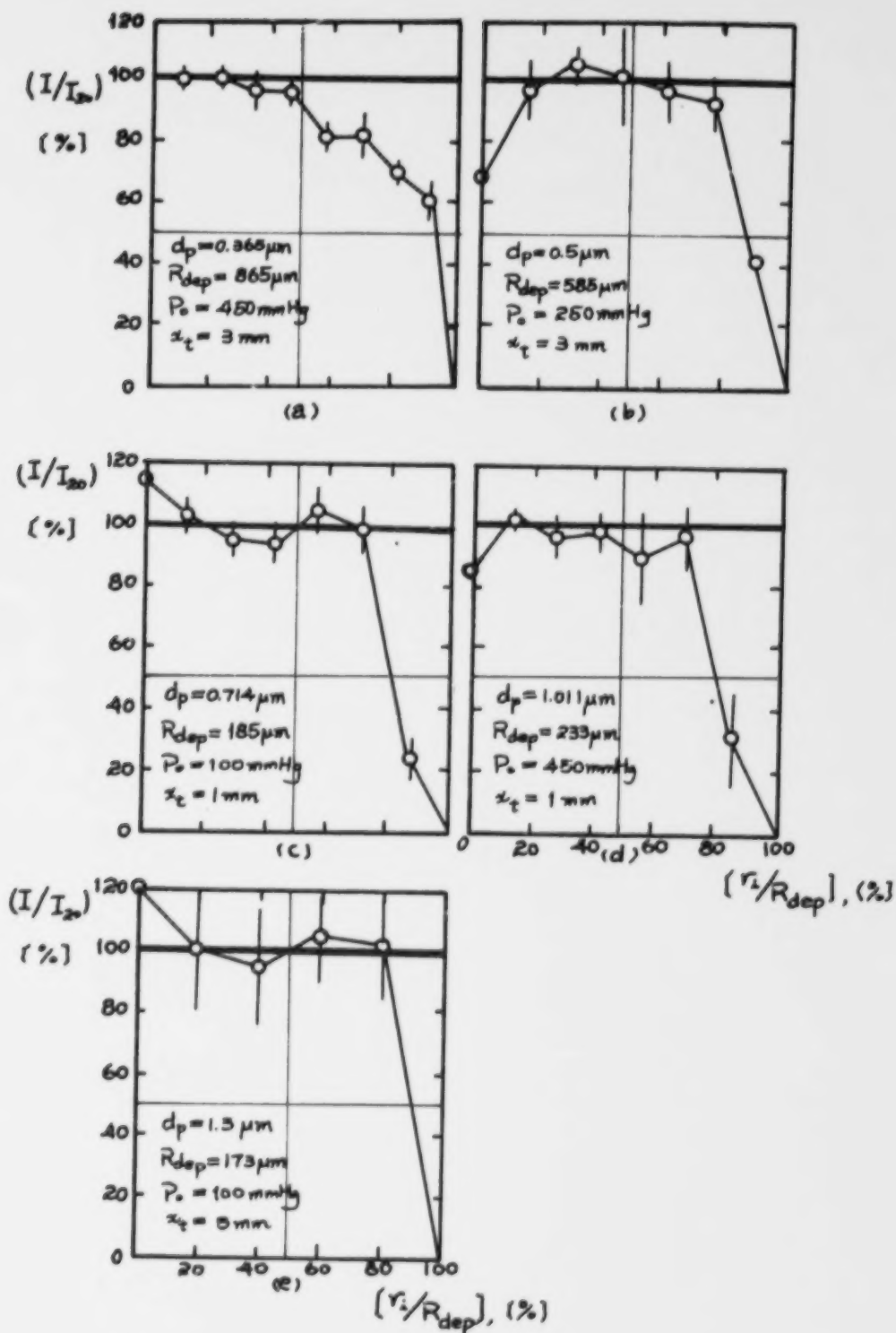


Fig. 20 Particle Density Distribution across the Deposit (error-bar representing the standard deviation of the data)

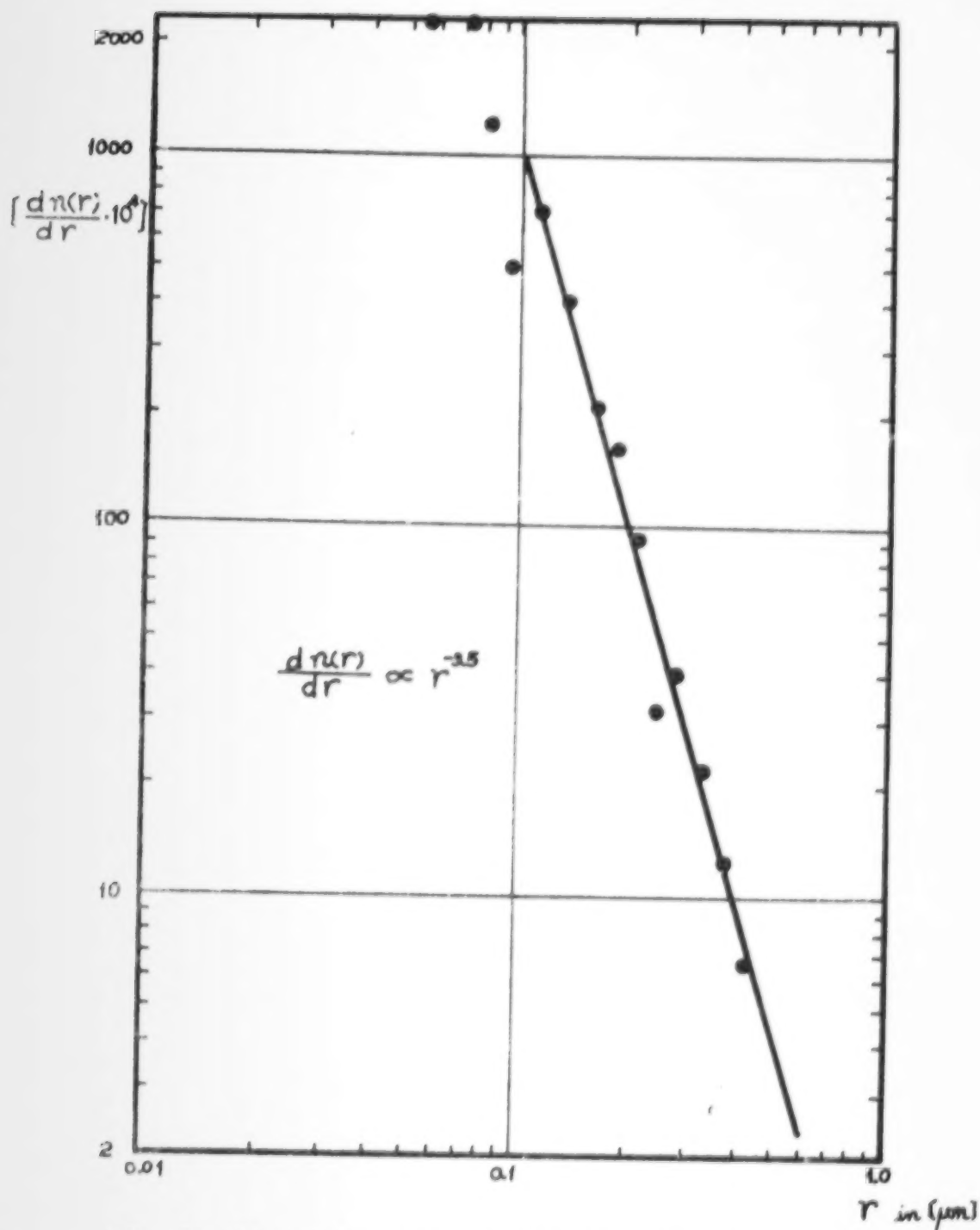


Fig. 21 Measured size distribution of an atmospheric aerosol collected in College Park, Md., based upon an assumed particle density of $\rho_p = 1.05 \text{ g/cm}^3$. Ordinate in arbitrary units.

curve Fig. 17. The size distribution given in Fig. 21 was then derived by differentiation. The data follow the so called Junge law and can be approximated by:

$$\frac{dn(r_p)}{dr_p} \propto (r_p)^{-3.5} \quad (51)$$

with r_p = particle radius.

The exponent of -3.5 is in agreement with Junge (1963) and many other authors who find exponents ranging from -3.5 to -4.5 for continental aerosols. The result indicates the potential of the particle beam method to deduce and measure size distributions of submicron aerosols.

Israel (1967) derived a semi-empirical relationship between the deposit diameter, the particle size and the source parameters given by:

$$\frac{D_{\text{dep}} - D^*}{x_t} = b \cdot \phi \quad (52)$$

with

$$\phi = \frac{1}{\rho_p d_p \cdot C_j}$$

where C_j is the Cunningham factor in the nozzle exit plane, and b is a constant which is determined by the nozzle geometry.

$$\frac{D_{\text{dep}} - D^*}{x_t} \text{ is plotted vs. } \phi \text{ in Fig. 22. The graph shows that}$$

the results can be expressed by Eq. (52) within about 15% for $\phi \geq 0.7 \cdot 10^{-8}$. This is in good agreement with the previously reported results where a capillary was used as nozzle (Israel, loc. cit.).

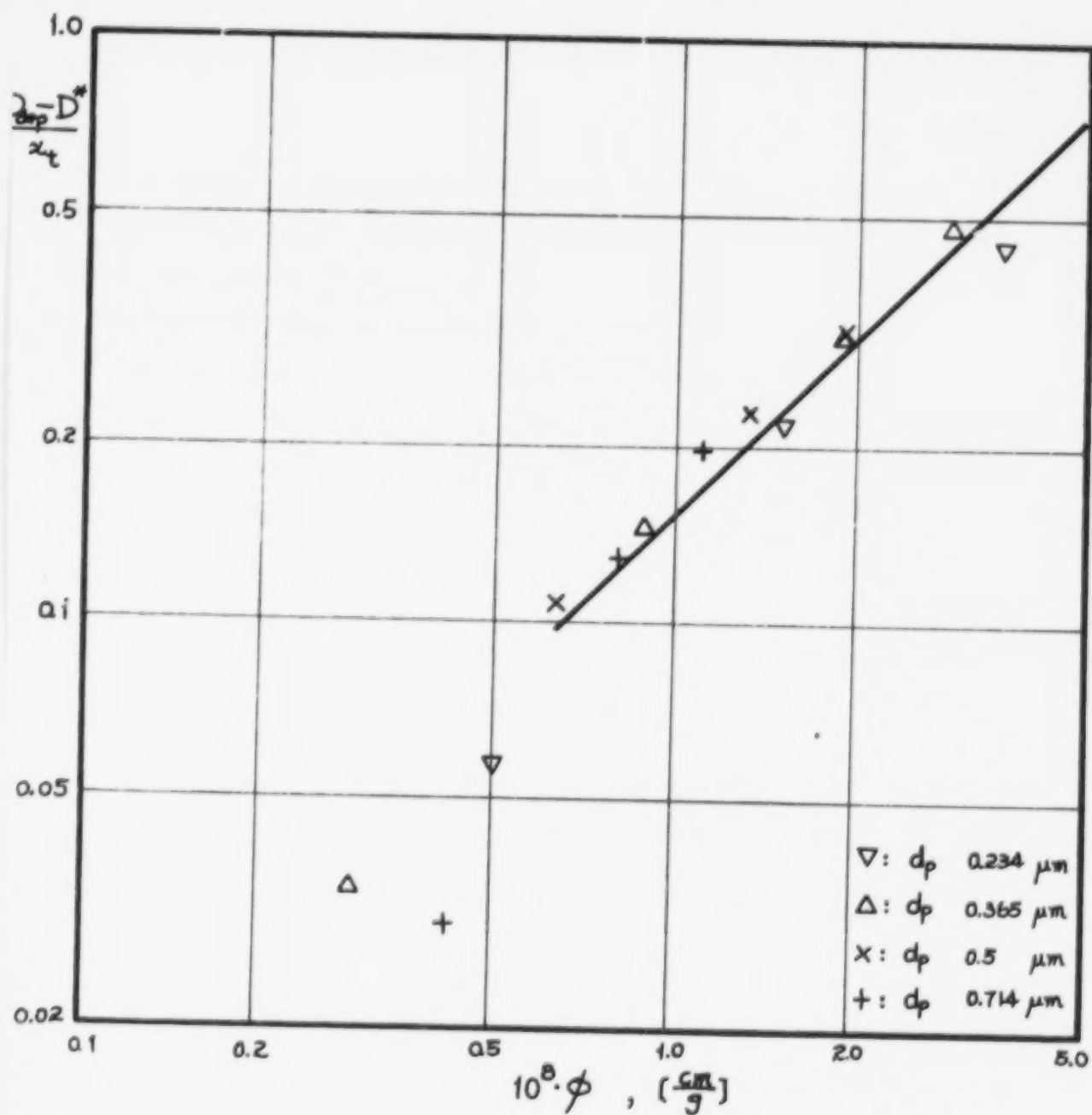


Fig. 22 Beam expansion $\frac{D_{Dep} - D^*}{x_t}$ as a function of ϕ

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A few experiments were carried out with nozzles of different geometries. The main characteristics of the beams produced by these nozzles are shortly discussed in the following section.

Nozzle B (Capillary)

Fig. 23 shows a typical deposit resulting from nozzle B. The circular deposit exhibits a ring-shaped area of very heavy deposit. Both the edge of the deposit and the ring follow the calculated trajectories and the d_j -values of the ring are directly related to the particle size as demonstrated by Fig. 24.

The characteristic o-ring shaped deposit is caused by the sharp edged nozzle inlet and the fact that the stop distance of the particles perpendicular to the nozzle axis (y-direction) can exceed the nozzle diameter. Fig. 25 gives an example of particle trajectories that were calculated for the inlet region and inside of Nozzle B using the relaxation method of Chapter I. The particle trajectories A, B, C cross each other in point P_1 near the nozzle inlet. A and B cross again near the nozzle outlet in P_2 while this double crossing does not occur for C. Thus the particles traveling in sector 5 will emerge from the nozzle in the small ring zone bounded by the trajectories A-B while the particles from sector 4 will be spread over the much larger zone A-C. The particles of sectors 1, 2, 3 either hit the opposite wall of the nozzle or emerge from zone C-D. The sector 6, 7, 8 particles are spread over B-O. The net effect is a maximum particle flux through zone A-B which results in the formation of a heavy ring-shaped deposit.

The sample calculation was made for $d_p = 0.365 \mu m$, $P_o = 250$ mmHg, and $P_e = 1$ mmHg. The final positions of particle 'a' and 'b' at the exit plane were $9 \mu m$ and $18 \mu m$ from the nozzle axis

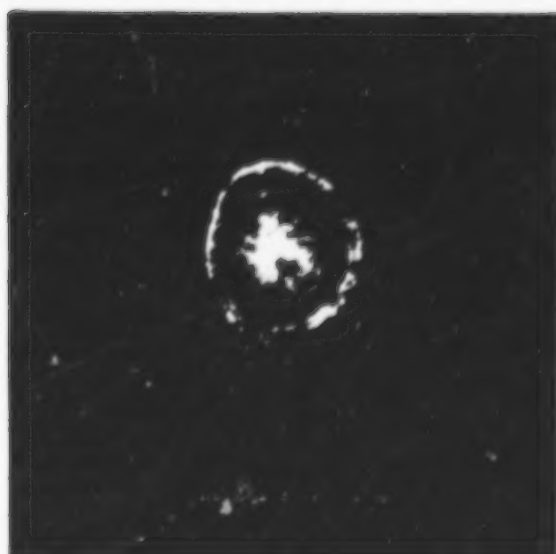


Fig. 23 Typical Deposit Formed by Nozzle B. Indicated scale corresponds to 200 μm .

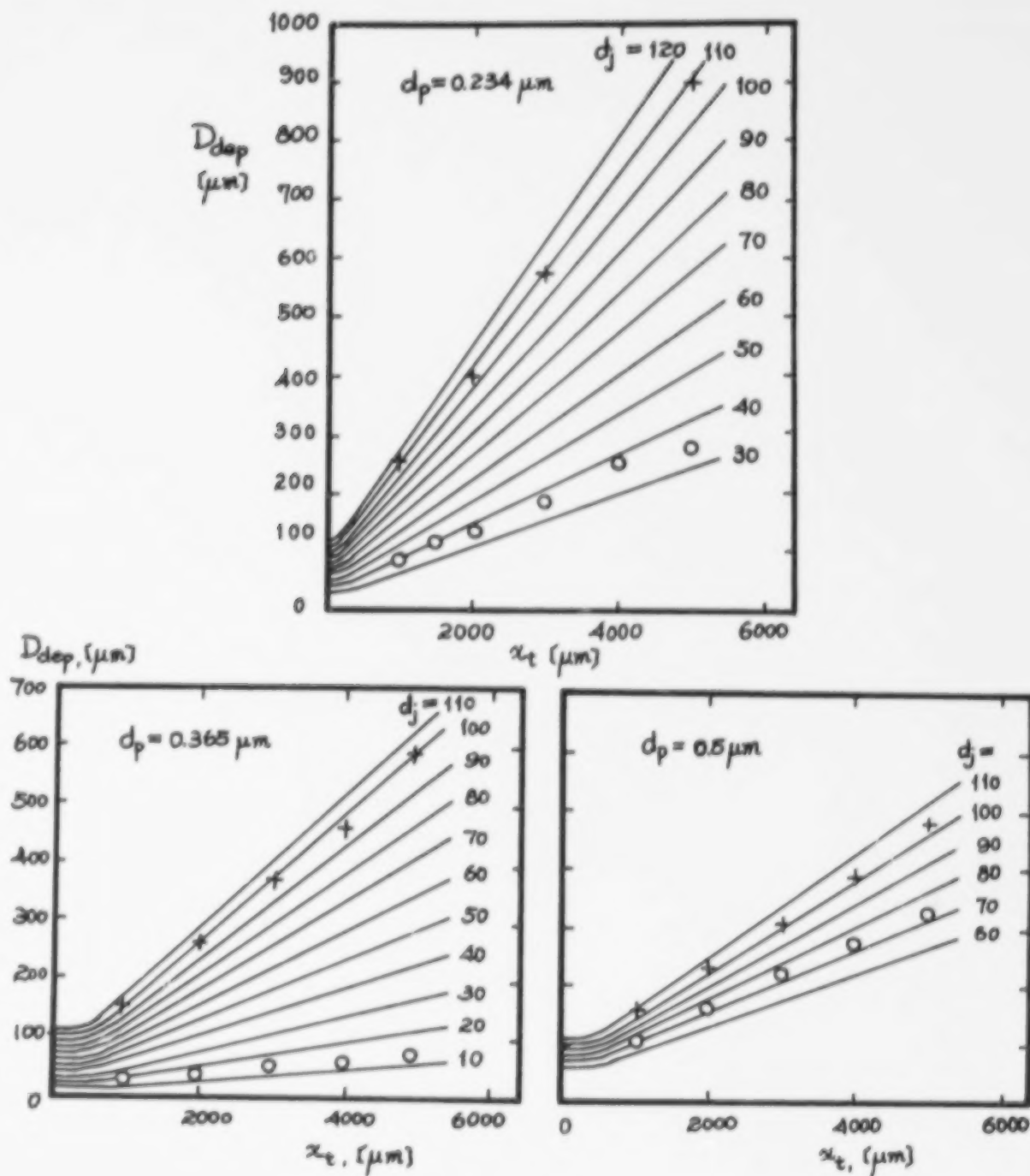


Fig. 24 Comparison Between Calculated and Measured Particle Deposit Diameters for Different Particle Sizes for Nozzle B ($p = 250 \text{ mmHg}$; +: edge of deposit; o: edge of ring-deposit)

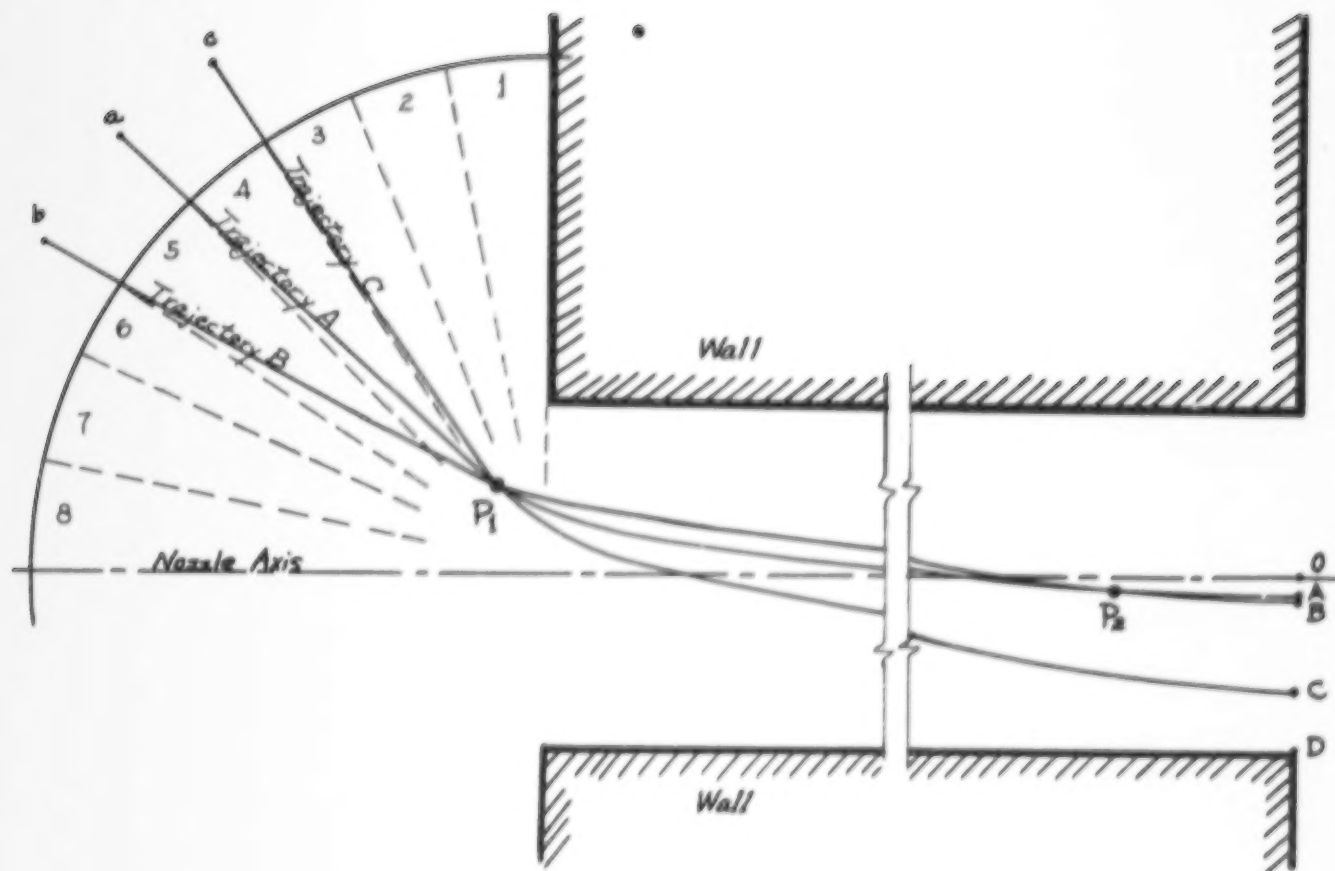


Fig. 25 Particle Trajectories at the Nozzle Inlet and inside Nozzle B
 ($d_p = 0.365\mu\text{m}$; $p_0 = 250\text{mmHg}$; $p_e = 1\text{mmHg}$)

respectively. Therefore, the radius or ring is about $r_j = 18 \mu\text{m}$. This agrees well with the experimental result which shows ring radius of about $15 \mu\text{m}$ (see Fig. 24).

There is another characteristic of the deposits formed by Nozzle B. Many deposits show a decrease in size with increasing source pressure until a minimum is reached at $P_o = (P_o)_{\min}$. Beyond this minimum, the deposit size increases with the source pressure. The occurrence of $(P_o)_{\min}$ can be explained as follows. For $P_o > (P_o)_{\min}$, the stop distance of the particles in y-direction is smaller than the nozzle radius, i.e., particles entering the upper half-section of the nozzle remain in this section and form a relative large circular deposit. If $P_o \approx (P_o)_{\min}$, the stop distance of particles is about the magnitude of the nozzle radius and some of the trajectories cross the nozzle axis. The result is a beam concentrated near the nozzle center which forms a deposit of minimum extent. For $P_o < (P_o)_{\min}$, the trajectories cross the nozzle axis and penetrate into the opposite half-section of the nozzle, therefore the beam width spreads again.

Hence, the stop distance, $l_1 = C \cdot \tau \cdot V_o$ in y-direction governs the minimum beam spread. Table 7 shows that $(P_o)_{\min}$ decreases with particle size and occurs for $l_1 = 57 \mu\text{m}$ which is about 20% less than the nozzle radius $R_N^* = 72.5 \mu\text{m}$.

Nozzle C (Convergent nozzle with constant cone angle $2\alpha_N = 10.9^\circ$)

The formation of particle beams with convergent nozzles was studied by Israel and Friedlander (1967) and Israel (1968). A main characteristic of these beams is the existence of a minimum for the beam width which is determined by source pressure and particle size. The mechanism for the constriction of the particle flow inside the nozzle

Table 7The Dependence of $(P_o)_{\min}$ on Particle Size (Nozzle B)

Particle size d_p (μm)	Measured $(P_o)_{\min}$ (mmHg)	Stop distance (μm)
0.5	600	55
0.365	250	57
0.234	100	60

is the same as was discussed for Nozzle B. Fig. 26 shows an example of the dependence of the beam width upon the square root of the source pressure for two different particle sizes.

The beam width shows a minimum for $0.365 \mu\text{m}$ particles at $(P_o)_{\text{min}} = 64 \text{ mmHg}$, while the minimum for $1.3 \mu\text{m}$ particles lies beyond $P_o = 760 \text{ mmHg}$. Near the minimum the beam width decreases or increases proportional to the square root of the source pressure.

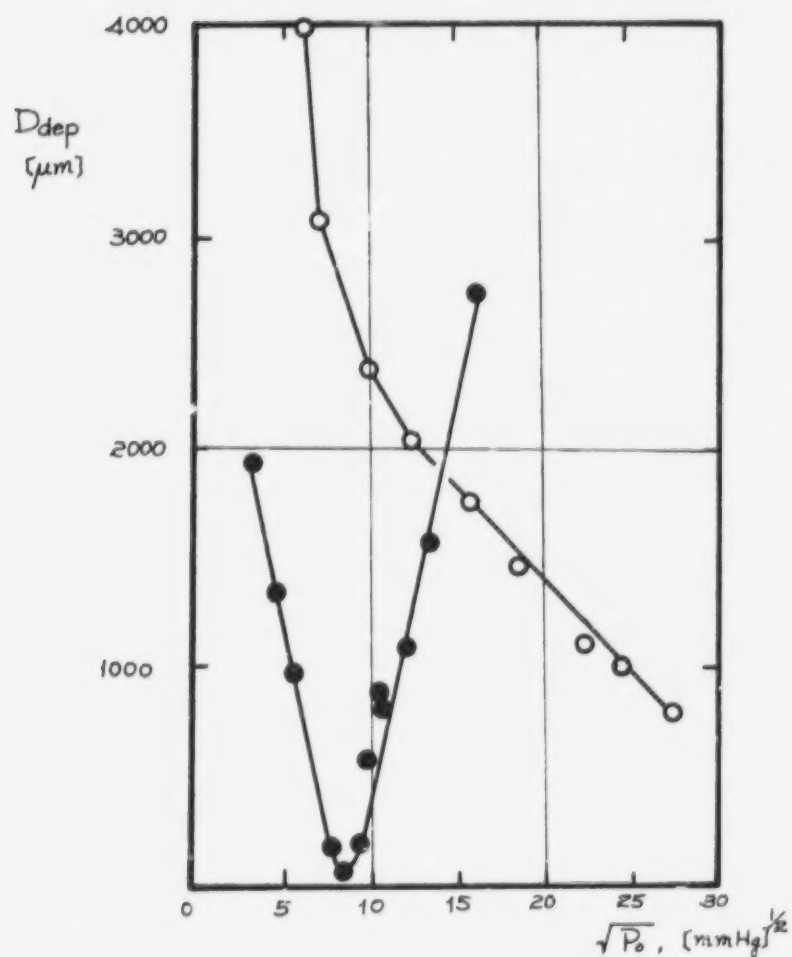


Fig. 26 The Size of Deposit vs. $\sqrt{P_0}$ for Different Particle Sizes

$$\left\{ \begin{array}{l} x_t = 10 \text{ mm} \\ \circ : d_p = 1.3 \mu\text{m} \\ \bullet : d_p = 0.365 \mu\text{m} \end{array} \right.$$

SUMMARY

Beams of submicron particles can be described adequately with the theory for two-dimensional, supersonic air flow with the assumption that the interaction between particles and gas jet consist of drag forces. The outlined mathematical procedures and computer programs can serve for the design of particle beams for special investigations.

Beams with rather uniform particle density can be generated with converging nozzles with the specifications that: (a) the particles are in velocity equilibrium with the gas flow inside the nozzle, and (b) the particles emerge from the nozzle parallel to the nozzle axis. For this nozzle the particle beam width is directly related to the particle size and increases with decreasing size. The described method allows the determination of the particle size of submicron aerosols with known density, from the width of their deposit with an accuracy of about 5% and can be adopted to measure size distributions of polydisperse aerosols in the range between 0.1 to 1.3 μm . It is likely that the range of application can be extended to smaller particle sizes with appropriate means for particle identification.

The particle flow in the beam is determined by the interaction between the gas flow and the particles in the vicinity of the nozzle inlet, inside the nozzle and downstream from the nozzle. Thus the nozzle geometry has a great impact upon the characteristic properties of the generated beam. Capillaries can be used to produce beams where the particles are heavily concentrated in small ring zones the diameter of

which is determined by the particle size. This property may be useful for partial separation of particles in a polydisperse aerosol.

For convergent nozzles, the beam width and therewith its intensity varies over orders of magnitude as a function of the source pressure with a maximum value at $(P_o)_{\min}$. This suggests the possibility of aerodynamically focusing particles of a selected size range in a polydisperse aerosol; the intensity of a selected particle size can be maximized by choosing the appropriate $(P_o)_{\min}$ source pressure for this size. By introducing a diaphragm into the beam, the core, which will be enriched with these particles, could be extracted and used to study physical or chemical properties of the submicron aerosol.

Acknowledgement

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APPENDIX A

COMPUTER PROGRAM FOR SUBSONIC
COMPRESSIBLE FLOW IN THE
NOZZLE INLET

Part One

Part Two

Program 2 Calculation of Stream-function for Subsonic Incompressible Flow

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'RUN PROJ1,304-02-120,WHANG,3,150/1500
'FOR,IS MAIN
  DIMENSION PSIA(100,100), RA(100,100), PSIB(100,100), RB(100,100)
  1 PSIC(100,50), RC(100,50)
  DO 100 I=1,71,10
  100 READ(1,105) (PSIA(I,J), J=1,71,10)
  105 FORMAT(8F10.0)
  BNDY=1.0
C RESIDUE AT BOUNDARY=0.00
C COURSE GRID NO.1
  I=1
  DO 110 J=1,71,10
  110 RA(I,J)=0.00
  I=71
  DO 115 J=1,71,10
  115 RA(I,J)=0.00
  J=1
  DO 120 I=1,71,10
  120 RA(I,J)=0.00
  J=71
  DO 125 I=1,71,10
  125 RA(I,J)=0.00
  N=1
C RELAXATION PATTERN AND RESIDUES IN COURSE GRID NO.1
  130 DO 135 I=11,61,10
  DO 135 J=11,61,10
  IA=I+10
  IB=I-10
  JA=J+10
  JB=J-10
  RA(I,J)=PSIA(IA,J)+PSIA(IB,J)+PSIA(I,JA)+PSIA(I,JB)-4.0*PSIA(I,J)
  135 PSIA(I,J)=PSIA(I,J)+1.0/4.0*RA(I,J)
  N=N+1
  IF(N.GT.5) GO TO 140
  GO TO 130
  140 DO 155 J=1,71,10
  WRITE(6,145) J
  145 FORMAT(1/5X, 'J=', I2, 5X, 'I=1,71,10')
  WRITE(6,150) (PSIA(I,J), I=1,71,10)
  150 FORMAT(1/5X, 8F10.5)
  155 WRITE(1,160) (PSIA(I,J), I=1,71,10)
  160 FORMAT(8F8.5)
C COURSE GRIDS NO. 2
  J=1
  DO 165 I=1,41,2
  165 PSIA(I,J)=0.00
  I=1
  DO 170 J=3,41,2
  170 PSIA(I,J)=BNDY
  DO 175 J=11,41,10
  DO 175 I=1,31,10
  DELPSI=1.0/5.0*(PSIA(I,J)-PSIA(I+10,J))
  DO 175 K=1,4,1
  KA=I+2*K
  CON=K
  175 PSIA(KA,J)=PSIA(I,J)-CON*DELPSI
  DO 180 I=11,41,10
  DO 180 J=1,31,10
  DELPSI=1.0/5.0*(PSIA(I,J+10)-PSIA(I,J))

```

```

DO 180 K=1,4,1
KA=J+2*K
CON=K
180 PSIA(I,KA)=PSIA(I,J)+CON*DELPSI
DO 185 J=3,39,2
DO 185 I=1,31,10
DELPSI=1.0/5.0*(PSIA(I,J)-PSIA(I+10,J))
DO 185 K=1,4,1
KA=I+2*K
CON=K
185 PSIA(KA,J)=PSIA(I,J)-CON*DELPSI
C RESIDUES ON BOUNDARY=0.00
I=1
DO 190 J=1,41,2
190 RA(I,J)=0.00
I=41
DO 195 J=1,41,2
195 RA(I,J)=0.00
J=1
DO 200 I=1,41,2
200 RA(I,J)=0.00
J=41
DO 205 I=1,41,2
205 RA(I,J)=0.00
C RESIDUES AND RELAXATION PATTERN IN COURSE GRIDS NO. 2
NN=1
210 DO 215 I=3,39,2
DO 215 J=3,39,2
IA=I+2
IB=I-2
JA=J+2
JB=J-2
RA(I,J)=PSIA(IA,J)+PSIA(IB,J)+PSIA(I,JA)+PSIA(I,JB)-4.0*PSIA(I,J)
215 PSIA(I,J)=PSIA(I,J)+1.0/4.0*RA(I,J)
NN=NN+1
IF(NN.GT.5) GO TO 220
GO TO 210
220 DO 235 J=1,41,2
WRITE(6,225) J
225 FORMAT(1/5X, 'J=', I2, 5X, 'I=1,41,2')
WRITE(6,230) (PSIA(I,J), I=1,41,2)
230 FORMAT(3(1/5X, 7F10.5))
235 WRITE(1,240) (PSIA(I,J), I=1,41,2)
240 FORMAT(3(1/7F10.5))
C COURSE GRIDS NO. 3
C INITIAL AND BOUNDARY VALUES OF PSIA(71,71)
J=1
DO 245 I=1,21,1
245 PSIA(I,J)=0.00
I=1
DO 250 J=2,21,1
250 PSIA(I,J)=BNDY
DO 255 J=3,21,2
DO 255 I=1,19,2
IA=I+1
255 PSIA(IA,J)=1.0/2.0*(PSIA(I,J)+PSIA(I+2,J))
DO 260 I=3,21,2
DO 260 J=1,19,2
JA=J+1

```

```

DO 180 K=1,4,1
KA=J+2*K
CON=K
180 PSIA(I,KA)=PSIA(I,J)+CON*DELPSI
DO 185 J=3,39,2
DO 185 I=1,31,10
DELPSI=1.0/5.0*(PSIA(I,J)-PSIA(I+10,J))
DO 185 K=1,4,1
KA=I+2*K
CON=K
185 PSIA(KA,J)=PSIA(I,J)-CON*DELPSI
C RESIDUES ON BOUNDARY=0.00
I=1
DO 190 J=1,41,2
190 RA(I,J)=0.00
I=41
DO 195 J=1,41,2
195 RA(I,J)=0.00
J=1
DO 200 I=1,41,2
200 RA(I,J)=0.00
J=41
DO 205 I=1,41,2
205 RA(I,J)=0.00
C RESIDUES AND RELAXATION PATTERN IN COURSE GRIDS NO. 2
NN=1
210 DO 215 I=3,39,2
DO 215 J=3,39,2
IA=I+2
IB=I-2
JA=J+2
JB=J-2
RA(I,J)=PSIA(IA,J)+PSIA(IB,J)+PSIA(I,JA)+PSIA(I,JB)-4.0*PSIA(I,J)
215 PSIA(I,J)=PSIA(I,J)+1.0/4.0*RA(I,J)
NN=NN+1
IF(NN.GT.5) GO TO 220
GO TO 210
220 DO 235 J=1,41,2
WRITE(6,225) J
225 FORMAT(1/5X, 'J=', I2, 5X, 'I=1,41,2')
WRITE(6,230) (PSIA(I,J), I=1,41,2)
230 FORMAT(3(1/5X, 7F10.5))
235 WRITE(1,240) (PSIA(I,J), I=1,41,2)
240 FORMAT(3(1/7F10.5))
C COURSE GRIDS NO. 3
C INITIAL AND BOUNDARY VALUES OF PSIA(71,71)
J=1
DO 245 I=1,21,1
245 PSIA(I,J)=0.00
I=1
DO 250 J=2,21,1
250 PSIA(I,J)=BNDY
DO 255 J=3,21,2
DO 255 I=1,19,2
IA=I+1
255 PSIA(IA,J)=1.0/2.0*(PSIA(I,J)+PSIA(I+2,J))
DO 260 I=3,21,2
DO 260 J=1,19,2
JA=J+1

```

```

260 PSIA(I,JA)=1.0/2.0*(PSIA(I,J)+PSIA(I,J+2))
    DO 265 J=2,20,1
    DO 265 I=1,19,2
    IA=I+1
265 PSIA(IA,J)=1.0/2.0*(PSIA(I,J)+PSIA(I+2,J))
C RESIDUES ON BOUNDARY =0.00
    I=1
    DO 270 J=1,21,1
270 RA(I,J)=0.00
    I=21
    DO 275 J=1,21,1
275 RA(I,J)=0.00
    J=1
    DO 280 I=1,21,1
280 RA(I,J)=0.00
    J=21
    DO 285 I=1,21,1
285 RA(I,J)=0.00
C RESIDUES AND RELAXATION PATTERN IN COURSE GRIDS NO. 3
    NNN=1
290 DO 295 I=2,20,1
    DO 295 J=2,20,1
    IA=I+1
    IB=I-1
    JA=J+1
    JB=J-1
    RA(I,J)=PSIA(IA,J)+PSIA(IB,J)+PSIA(I,JA)+PSIA(I,JB)-4.0*PSIA(I,J)
295 PSIA(I,J)=PSIA(I,J)+1.0/4.0*RA(I,J)
    NNN=NNN+1
    IF(NNN.GT.5) GO TO 300
    GO TO 290
300 DO 310 J=1,21,1
    WRITE(6,305) J
305 FORMAT(1/5X, 'J=', I2, 5X, 'I=1,21,1')
    WRITE(6,310) (PSIA(I,J), I=1,21,1)
310 FORMAT(3(1/5X, 7F10.5))
315 WRITE(1,320) (PSIA(I,J), I=1,21,1)
320 FORMAT(3(1/7F8.5))
C IN FINER GRIDS
C TRANSFER THE VALUES OF STREAM-FUNCTION FROM COURSE GRIDS INTO FINE GRIDS.
    DO 325 I=1,11,1
    DO 325 J=1,11,1
    IT=5*I-4
    JT=5*J-4
325 PSIB(IT,JT)=PSIA(I,J)
C INITIAL AND BOUNDARY VALUES OF PSIC(I,J) IN NOZZLE DUCT
    I=1
    DO 330 J=1,6,1
330 READ(5,335) PSIC(I,J)
335 FORMAT(F10.0)
    DO 340 I=2,51,1
    DO 340 J=1,6,1
340 PSIC(I,J)=PSIC(1,J)
C INITIAL AND BOUNDARY VALUES OF PSIB(I,J) IN NOZZLE INLET REGION
    J=1
    DO 345 I=1,51,1
345 PSIB(I,J)=0.00
    I=1
    DO 350 J=6,51,1

```

```

350 PSIB(I,J)=BNDY
DO 355 J=6,51,5
DO 355 I=1,46,5
DELPSI=1.0/5.0*(PSIB(I,J)-PSIB(I+5,J))
DO 355 K=1,4,1
KA=I+K
CON=K
355 PSIB(KA,J)=PSIB(I,J)-CON*DELPSI
DO 360 I=6,51,5
DO 360 J=1,46,5
DELPSI=1.0/5.0*(PSIB(I,J+5)-PSIB(I,J))
DO 360 K=1,4,1
KA=J+K
CON=K
360 PSIB(I,KA)=PSIB(I,J)+CON*DELPSI
C OVERLAPED LINE
DO 365 J=2,5,1
365 PSIB(1,J)=PSIB(51,J)
DO 370 J=2,50,1
DO 370 I=1,46,5
DELPSI=1.0/5.0*(PSIB(I,J)-PSIB(I+5,J))
DO 370 K=1,4,1
KA=I+K
CON=K
370 PSIB(KA,J)=PSIB(I,J)-CON*DELPSI
C RESIDUES ON BOUNDARY= 0.00 IN NOZZLE DUCT.
I=1
DO 375 J=2,5,1
375 RC(I,J)=0.00
J=1
DO 380 I=1,51,1
380 RC(I,J)=0.00
J=6
DO 385 I=1,51,1
385 RC(I,J)=0.00
C RESIDUES ON BOUNDARY =0.00 IN NOZZLE INLET REGION
I=1
DO 390 J=6,51,1
390 RB(I,J)=0.00
I=51
DO 395 J=1,51,1
395 RB(I,J)=0.00
J=1
DO 400 I=1,51,1
400 RB(I,J)=0.00
J=51
DO 405 I=1,51,1
405 RB(I,J)=0.00
C RESIDUES AND RELAXATION PATTERN IN NOZZLE DUCT
NA=1
410 DO 420 I=2,51,1
DO 420 J=2,5,1
IF(I.EQ.51) GO TO 415
IA=I+1
IB=I-1
JA=J+1
JB=J-1
RC(I,J)=PSIC(IA,J)+PSIC(IB,J)+PSIC(I,JA)+PSIC(I,JB)
PSIC(I,J)=1.0/4.0*RC(I,J)

```

```

GO TO 420
415 RC(I,J)=PSIC(I-1,J)+PSIC(I,J+1)+PSIC(I,J-1)+PSIB(2,J)
    PSIC(I,J)=1.0/4.0*RC(I,J)
420 CONTINUE
C OVERLAPED LINE
    I=1
    DO 425 J=2,5,1
425 PSIB(1,J)=PSIC(51,J)
C RESIDUES AND RELAXATION PATTERN IN NOZZLE INLET REGION
    DO 430 I=2,50,1
    DO 430 J=2,50,1
    IA=I+1
    IB=I-1
    JA=J+1
    JB=J-1
    RB(I,J)=PSIB(IA,J)+PSIB(IB,J)+PSIB(I,JA)+PSIB(I,JB)
430 PSIB(I,J)=1.0/4.0*RB(I,J)
    NA=NA+1
    IF(NA.GT.10) GO TO 435
    GO TO 410
435 DO 450 I=1,51,1
    WRITE(6,440) I
440 FORMAT(/5X, 'I=', I2, 5X, 'J=1,6,1')
    WRITE(6,445) (PSIC(I,J), J=1,6,1)
445 FORMAT(/5X, 6F10.5)
450 WRITE(1,455) (PSIC(I,J), J=1,6,1)
455 FORMAT(6F8.5)
    DO 470 I=1,51,1
    WRITE(6,460) I
460 FORMAT(/5X, 'I=', I2, 5X, 'J=1,51,1')
    WRITE(6,465) (PSIB(I,J), J=1,51,1)
465 FORMAT(/5X, 3F10.5, /6(/5X, 8F10.5))
470 WRITE(1,475) (PSIB(I,J), J=1,51,1)
475 FORMAT(3F8.5/, 6(/8F8.5))
END

```

```

'XQT
+0.00      +1.00      +1.00      +1.00      +1.00      +1.00      +1.00      +1.00
+0.00      +0.91      +0.92      +0.93      +0.94      +0.96      +0.99      +1.00
+0.00      +0.80      +0.82      +0.835     +0.87      +0.90      +0.985     +1.00
+0.00      +0.60      +0.70      +0.76      +0.84      +0.90      +0.98      +1.00
+0.00      +0.40      +0.60      +0.68      +0.81      +0.895     +0.975     +1.00
+0.00      +0.30      +0.50      +0.65      +0.78      +0.89      +0.97      +1.00
+0.00      +0.2135     +0.4168     +0.6031     +0.7629     +0.8884     +0.9700     +1.00
+0.00      +0.2135     +0.4168     +0.6031     +0.7629     +0.8884     +0.9700     +1.00
+0.00
+0.2960
+0.5680
+0.7920
+0.9440
+1.0000

```

```

425 PSIB(I,J)=PSIC(51,J)
C RESIDUES AND RELAXATION PATTERN IN NOZZLE INLET REGION
DO 430 I=2,50,1
DO 430 J=2,50,1
IA=I+1
IB=I-1
JA=J+1
JB=J-1
RB(I,J)=PSIB(IA,J)+PSIB(IB,J)+PSIB(I,JA)+PSIB(I,JB)
430 PSIB(I,J)=1.0/4.0*RB(I,J)
NA=NA+1
IF(NA,3T,10) GO TO 435
GO TO 410
435 DO 450 I=1,51,1
WRITE(6,440) I
440 FORMAT(15X, 'I=', I2, 5X, 'J=1,6,1')
WRITE(6,445) (PSIC(I,J), J=1,6,1)
445 FORMAT(15X, 6F10.5)
450 WRITE(1,455) (PSIC(I,J), J=1,6,1)
455 FORMAT(6F8.5)
DO 470 I=1,51,1
WRITE(6,460) I
460 FORMAT(15X, 'I=', I2, 5X, 'J=1,51,1')
WRITE(6,465) (PSIB(I,J), J=1,51,1)
465 FORMAT(15X, 3F10.5, /6(15X, 8F10.5))
470 WRITE(1,475) (PSIB(I,J), J=1,51,1)
475 FORMAT(3F8.5/, 6(18F8.5))
END

```

'XGT

+0.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00
+0.00	+0.91	+0.92	+0.93	+0.94	+0.96	+0.99	+1.00
+0.00	+0.80	+0.82	+0.835	+0.87	+0.90	+0.985	+1.00
+0.00	+0.60	+0.70	+0.76	+0.84	+0.90	+0.98	+1.00
+0.00	+0.40	+0.60	+0.68	+0.81	+0.895	+0.975	+1.00
+0.00	+0.30	+0.50	+0.65	+0.78	+0.89	+0.97	+1.00
+0.00	+0.2135	+0.4168	+0.6031	+0.7629	+0.8884	+0.9700	+1.00
+0.00	+0.2135	+0.4168	+0.6031	+0.7629	+0.8884	+0.9700	+1.00

+0.00

+0.2960

+0.5680

+0.7920

+0.9440

+1.0000

'FIN

Part 2 Calculation of stream-function for subsonic Compressible Flow

```

*RUN PROJ1,304-02-120,WHANG,2,150/1000
*FOR,IS MAIN
C DIMENSION PO(ATM), POA(MEGA-DYNE/CM SQUARE), TEMP(ABS.), DENSO(GM/LITER),
C DD(MICRONS), SUND0(CM/SEC), DELTA(MICRONS), BNDY( DIMENSIONLESS), W(CM-
C MICRON SQUARE/SEC), CON(DIMENSIONLESS)
    DIMENSION PSI(51,51), PSIN(51,6), RB(51,51), RC(51,6),
    1 QN(51,6), Q(51,51), R(51,51), RN(51,6), DC(51,51), DON(51,6),
    1 DN(51,6), D(51,51)
    PO=+760.0
    POA=PO/760.0*1.013
    TEMPO=+298.0
    DENSO=1.185
    DD=+1.0.0
    SUND0=2000.0*SQRT(TEMPO)
    DELTA=+17.0
    BNDY=1.0000
    PAI=+3.14159
    GAMMA=1.4
    GAM=2.0/(GAMMA+1.0)
    W=PAI*(DD/2.0)**(2.0)*POA/DENSO*SQRT(GAMMA/GAM*0.336/TEMPO)*
    1 GAM**((1.0/(GAMMA-1.0))*10.0**(+6.0)
    DO 300 I=1,51,1
300 READ(5,1) (PSIN(I,J), J=1,6,1)
    1 FORMAT(6F8.5)
    DO 305 I=1,51,1
305 READ(5,2) (PSI(I,J), J=1,51,1)
    2 FORMAT(3F8.5/, 6(/8F8.5))
    ITER=1
C RESIDUES ON BOUNDARY =0.00 IN NOZZLE PART
370 I=1
    DO 375 J=2,5,1
375 RC(I,J)=0.00
    J=1
    DO 380 I=1,51,1
380 RC(I,J)=0.00
    J=6
    DO 385 I=1,51,1
385 RC(I,J)=0.00
C RESIDUES ON BOUNDARY =0.00 IN NOZZLE INLET REGION
    I=1
    DO 390 J=6,51,1
390 RB(I,J)=0.00
    I=51
    DO 395 J=1,51,1
395 RB(I,J)=0.00
    J=1
    DO 400 I=1,51,1
400 RB(I,J)=0.00
    J=51
    DO 405 I=1,51,1
405 RB(I,J)=0.00
C RESIDUES AND RELAXATION PATTERN IN NOZZLE DUCT
    NA=1
410 DO 420 I=2,51,1
    DO 420 J=2,5,1
    IF(I.EQ.51) GO TO 415
    IA=I+1
    IB=I-1
    JA=J+1

```

```

      JB=J-1
      RC(I,J)=PSIN(IA,J)+PSIN(IB,J)+PSIN(I,JA)+PSIN(I,JB)
      PSIN(I,J)=1.0/4.0*RC(I,J)
      GO TO 420
415  RC(I,J)=PSIN(I-1,J)+PSIN(I,J+1)+PSIN(I,J-1)+PSI(2,J)
      PSIN(I,J)=1.0/4.0*RC(I,J)
420  CONTINUE
C OVERLAPED LINE
      I=1
      DO 425 J=2,5,1
425  PSI(I,J)=PSIN(51,J)
C RESIDUES AND RELAXATION PATTERN IN NOZZLE INLET REGION
      DO 430 I=2,50,1
      DO 430 J=2,50,1
      IA=I+1
      IB=I-1
      JA=J+1
      JB=J-1
      RB(I,J)=PSI(IA,J)+PSI(IB,J)+PSI(I,JA)+PSI(I,JB)
430  PSI(I,J)=1.0/4.0*RB(I,J)
      NA=NA+1
      IF(NA.GT.3) GO TO 435
      GO TO 410
435  J=1
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 510 I=2,51,1
510  QN(I,J)=CON*2.0*PSIN(I,J+1)
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 515 J=2,5,1
      DO 515 I=2,50,1
      A=(PSIN(I-1,J)-PSIN(I+1,J))*2.0
      B=(PSIN(I,J+1)-PSIN(I,J-1))*2.0
515  QN(I,J)=CON*SQRT(A+B)
      J=6
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 520 I=2,50,1
520  QN(I,J)=2.0*CON*(BNDY-PSIN(I,J-1))
      J=6
      CON=W/(DD*SUNDO*2.0*DELTA)
      I=51
      A=(BNDY-PSIN(I,J-1))*2.0
      B=((PSIN(I-1,J)-PSI(2,J))/2.0)*2.0
      QN(I,J)=CON*SQRT(A+B)
      I=51
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 525 J=2,5,1
      A=(PSIN(51,J+1)-PSIN(51,J-1))*2.0
      B=(PSIN(50,J)-PSI(2,J))*2.0
525  QN(I,J)=CON*SQRT(A+B)
C Q IN INLET PART
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 530 I=2,50,1
      DO 530 J=2,50,1
      A=(PSI(I,J+1)-PSI(I,J-1))*2.0
      B=(PSI(I-1,J)-PSI(I+1,J))*2.0
530  Q(I,J)=CON*SQRT(A+B)
      J=1
      CON=W/(DD*SUNDO*2.0*DELTA)
      DO 535 I=2,50,1

```

```

535 Q(I,J)=CON*2.0*PSI(I,J+1)
    I=1
    DO 540 J=7,50,1
540 Q(I,J)=CON*2.0*(BNDY-PSI(I+1,J))
    I=1
    DO 545 J=1,6,1
545 Q(I,J)=QN(51,J)
C DGN IN NOZZLE
    DO 550 I=2,50,1
    DO 550 J=2,5,1
    A=(PSIN(I,J+1)-PSIN(I,J-1))**(2.0)
    B=(PSIN(I+1,J)-PSIN(I-1,J))**(2.0)
550 DGN(I,J)=QN(I,J)*SQRT(A+B)
    J=1
    DO 555 I=2,51,1
555 DGN(I,J)=QN(I,J)*2.0*PSIN(I,J+1)
    J=6
    DO 560 I=2,50,1
560 DGN(I,J)=QN(I,J)*2.0*(BNDY-PSIN(I,J-1))
    I=51
    DO 565 J=2,5,1
    A=(PSIN(I,J+1)-PSIN(I,J-1))**(2.0)
    B=(PSIN(I-1,J)-PSIN(I+1,J))**(2.0)
565 DGN(I,J)=QN(I,J)*SQRT(A+B)
    J=6
    I=51
    A=(BNDY-PSIN(I,J-1))**(2.0)
    B=(BNDY-PSI(I,6))**(2.0)
    DGN(I,J)=QN(I,J)*SQRT(A+B)
C DO IN INLET PART
C OVERLAPED AREA
    I=1
    DO 570 J=1,6,1
570 DQ(I,J)=DGN(51,J)
    I=1
    DO 575 J=7,50,1
575 DQ(I,J)=Q(I,J)*2.0*(BNDY-PSI(I+1,J))
    J=1
    DO 580 I=2,50,1
580 DQ(I,J)=Q(I,J)*2.0*PSI(I,J+1)
    DO 585 I=2,50,1
    DO 585 J=2,50,1
    A=(PSI(I,J+1)-PSI(I,J-1))**(2.0)
    B=(PSI(I-1,J)-PSI(I+1,J))**(2.0)
585 DQ(I,J)=Q(I,J)*SQRT(A+B)
C USING ITERATION METHOD TO FIND DIMENSIONLESS DENSITY RATIOS
C DN IN NOZZLE PART
    DO 595 I=2,51,1
    DO 595 J=1,6,1
    N=1
    DN(I,J)=0.70
590 AA=DN(I,J)**(2.0)*(1.0-DN(I,J))
    BB=DGN(I,J)**(2.0)*0.2*(1.0-DN(I,J))**(0.6)
    CC=2.0*DN(I,J)-1.6*DN(I,J)**(2.0)
    DNNIW=DN(I,J)-(AA-BB)/CC
    IF(ABS(DNNEW-DN(I,J)).LT.0.001) GO TO 595
    N=N+1
    IF(N.GT.15) GO TO 595
    DN(I,J)=DNNEW

```

```

      GO TO 590
595 CONTINUE
C D IN INLET PART
  I=1
  DO 600 J=1,6,1
600 D(I,J)=DN(51,J)
  DO 610 I=1,50,1
  DO 610 J=1,50,1
    IF(J.LE.6.AND.I.EQ.1) GO TO 610
    N=1
    D(I,J)=0.80
605 AA=D(I,J)**(2.0)*(1.0-D(I,J))
    BB=DQ(I,J)**(2.0)*0.2*(1.0-D(I,J))**(0.6)
    CC=2.0*D(I,J)-1.6*D(I,J)**(2.0)
    DNEW=D(I,J)-(AA-BB)/CC
    IF(ABS(DNEW-D(I,J)).LT.0.001) GO TO 610
    N=N+1
    IF(N.GT.15) GO TO 610
    D(I,J)=DNEW
    GO TO 605
610 CONTINUE
C CALCULATION OF RESIDUE R(51,51), RN(51,6)
C RN IN NOZZLE PART
  DO 615 I=3,50,1
  DO 615 J=2,5,1
    A1=ALOG(DN(I-1,J))
    A2=ALOG(DN(I+1,J))
    A3=ALOG(DN(I,J+1))
    A4=ALOG(DN(I,J-1))
615 RN(I,J)=1.0/4.0*((PSIN(I-1,J)-PSIN(I+1,J))*(A1-A2)
  1 +(PSIN(I,J+1)-PSIN(I,J-1))*(A3-A4))
    I=51
    DO 620 J=2,5,1
    A1=ALOG(DN(I-1,J))
    A2=ALOG(D(2,J))
    A3=ALOG(DN(I,J+1))
    A4=ALOG(DN(I,J-1))
620 RN(I,J)=1.0/4.0*((PSIN(I-1,J)-PSI(2,J))*(A1-A2)
  1 +(PSIN(I,J+1)-PSIN(I,J-1))*(A3-A4))
C R IN INLET PART
C OVERLAPED PART
  I=1
  DO 625 J=2,5,1
625 R(I,J)=RN(51,J)
  DO 630 I=2,49,1
  DO 630 J=2,49,1
    A1=ALOG(D(I-1,J))
    A2=ALOG(D(I+1,J))
    A3=ALOG(D(I,J+1))
    A4=ALOG(D(I,J-1))
630 R(I,J)=1.0/4.0*((PSI(I-1,J)-PSI(I+1,J))*(A1-A2)
  1 +(PSI(I,J+1)-PSI(I,J-1))*(A3-A4))
C LIQUIDATION PROCESSES
C LIQUIDATION IN NOZZLE PART, PSIN(I,J)
  DO 635 I=3,51,1
  DO 635 J=2,5,1
635 PSIN(I,J)=PSIN(I,J)+1.0/4.0*(RN(I,J)+RN(I,J-1)+RN(I,J+1)+RN(I+1,J)
  1+RN(I-1,J))
C LIQUIDATION IN INLET PART, PSI(I,J)

```

C OVERLAPED AREA

```

      I=1
      DO 640 J=2,5,1
640   PSI(I,J)=PSIN(51,J)
      DO 645 I=2,49,1
      DO 645 J=2,49,1
645   PSI(I,J)=PSI(I,J)+1.0/4.0*(R(I,J)+R(I,J-1)+R(I,J+1)
      1 +R(I+1,J)+R(I-1,J))
      ITER=ITER+1
      IF(ITER.GT.10) GO TO 650
      GO TO 370
650   DO 655 I=1,51,1
      WRITE(6,5) I
      5   FORMAT(/5X, 'I=', I2, 5X, 'J=1,6,1')
      WRITE(6,6) (PSIN(I,J), J=1,6,1)
      6   FORMAT(/5X, 6F10.5)
655   WRITE(1,7) (PSIN(I,J), J=1,6,1)
      7   FORMAT(6F8.5)
      DO 660 I=1,51,1
      WRITE(6,8) I
      8   FORMAT(/5X, 'I=', I2, 5X, 'J=1,51,1')
      WRITE(6,9) (PSI(I,J), J=1,51,1)
      9   FORMAT(/5X, 3F10.5, /6(/5X, 8F10.5))
660   WRITE(1,10) (PSI(I,J), J=1,51,1)
      10  FORMAT(3F8.5/, 6(/8F8.5))
      DO 670 I=1,51,1
      WRITE(6,5) I
      WRITE(6,6) (DN(I,J), J=1,6,1)
670   WRITE(1,7) (DN(I,J), J=1,6,1)
      DO 675 I=1,51,1
      WRITE(6,8) I
      WRITE(6,9) (D(I,J), J=1,51,1)
675   WRITE(1,10) (D(I,J), J=1,51,1)
      END

```

```

'XQT
'FIN
''

```

```

''
NIF'

```

LIST OF SYMBOLS

Part 1 Sonic Incompressible Flow

BNDY	Boundary value of stream function, η (dimensionless)
PSIA, PSIB, PSIC	η values in grid A, B, and C respectively (dimensionless)
RA, RB, RC	Residual at nodes of grid A, B, and C respectively (Dimensionless)

Part 2 Sonic Compressible Flow

D	(ρ/ρ_0) near the nozzle inlet
DD	D^* in (μm)
DELTA	δ in (μm)
DENSØ	ρ_0 in (g/l)
DN	(ρ/ρ_0) in the nozzle duct
DQ	M near nozzle inlet
DQN	M inside the nozzle
Q	q_1 in (cm/sec) near nozzle inlet
QN	q_1 in (cm/sec) inside the nozzle
R	The residual of compressible flow inside the nozzle
RB	The residual of incompressible flow near the nozzle inlet
RC	The residual of incompressible flow near the nozzle inlet
RN	The residual of compressible flow inside the nozzle
PØ	P_0 in (atm.)
PØA	P_0 in (mega-dyns/cm ²)
PSI, PSIN	ψ near the nozzle inlet and inside the nozzle respectively

SUNDQ

 c_0 in (cm/sec)

TEMP

T in ($^{\circ}$ K)

W

The flow rate in (cm- μ m²/ sec)

APPENDIX B

COMPUTER PROGRAM FOR SUPERSONIC
UNDEREXPANDED AIR JET

```

'RUN PROJ1,304-02-120,WHANG,2,500/2500
'FOR,IS MAIN
C PROGRAM OF FLOW FIELDS OF FREE UNDEREXPENSED JETS.
C
C CALCULATE THE NUMBER LIMIT, LIM, FOR THE FLOW FIELD AT DIFFERENT RATIO
C OF UPSTREAM AND DOWSTREAM PRESSURES.
  DIMENSION I1(27,90), I2(27,90), M(27,90), W(27,90), O(27,90),
  1AL(27,90), OAL1(27,90), OAL2(27,90), DP(27,90), DT(27,90),
  2DD(27,90), MS(27,90), SLOP1(27,90), SLOP2(27,90)
  DIMENSION X(27,90), Y(27,90)
  REAL I1, I2, M, MS
  REAL MACH, MACHS
50 READ(5,101) M(1,1), O(1,1), PO, PE
101 FORMAT(F10.0, F10.0, F10.0, F10.0)
  PEPO=PE/PO
  PI=180.0/3.14159
  MACH=(5.0*(PEPO**(-0.2857)-1.0))**(0.5)
  MACHS=SQRT(1.2*MACH**2/(1.0+0.2*MACH**2))
  ABCD=((MACHS**2-1.0)/(1.0-MACHS**2/6.0))**(0.5)
  OMAGA=((6.0**0.5)*ATAN(ABCD/(6.0**0.5))-ATAN(ABCD))*PI
  THETA=-OMAGA
  LIM=OMAGA
  M(1,LIM)=MACH
C
C COMPUTATION OF FLOW FIELD NO.(1,1)
  P=M(1,1)**2
  MS(1,1)=SQRT(1.2*P/(1.0+0.2*P))
  Q=MS(1,1)**2
  R=SQRT((Q-1.0)/(1.0-Q/6.0))
  W(1,1)=(6.0**0.5*ATAN(R/6.0**0.5)-ATAN(R))*PI
  I1(1,1)=0.5*(1000.0+O(1,1)+W(1,1))
  I2(1,1)=0.5*(1000.0+O(1,1)-W(1,1))
C
C COMPUTATION OF FLOW FIELDS FROM NO.(1,2) TO NO.(1,LIM-1)
  LIMA=LIM-1
  DO 201 K=2,LIMA
    I1(1,K)=I1(1,K-1)
    AK=K
    O(1,K)=(AK-1.0)*(-1.0)
    I2(1,K)=1000.0+O(1,K)-I1(1,K)
201 W(1,K)=2.0*I1(1,K)-1000.0-O(1,K)
C
C COMPUTATION OF FLOW FIELD NO(1,LIM)
  P=M(1,LIM)**2
  MS(1,LIM)=SQRT(1.2*P/(1.0+0.2*P))
  Q=MS(1,LIM)**2
  R=SQRT((Q-1.0)/(1.0-Q/6.0))
  W(1,LIM)=(6.0**0.5*ATAN(R/6.0**0.5)-ATAN(R))*PI
  I1(1,LIM)=I1(1,1)
  I2(1,LIM)=I1(1,LIM)-W(1,LIM)
  O(1,LIM)=2.0*I1(1,LIM)-1000.0-W(1,LIM)
C COMPUTATION OF FLOW FIELDS FROM NO(2,1) TO NO.(27,LIM)
  DO 202 L=2, 27
    DO 202 K=1, LIM
      IF(K.EQ.1) GO TO 203
      IF(K.EQ.LIM) GO TO 204
      I1(L,K)=I1(L, K-1)
      I2(L,K)=I2(L-1, K+1)

```

```

O(L,K)=I1(L,K)+I2(L,K)-1000.0
W(L,K)=I1(L,K)-I2(L,K)
GO TO 202
203 I2(L,1)=I2(L-1,2)
O(L,1)=0.0
W(L,1)=1000.0-2.0*I2(L,1)+O(L,1)
I1(L,1)=I2(L,1)+W(L,1)
GO TO 202
204 M(L,LIM)=M(1,LIM)
W(L,LIM)=W(1,LIM)
I1(L,LIM)=I1(L,LIM-1)
I2(L,LIM)=I1(L,LIM)-W(L,LIM)
O(L,LIM)=2.0*I1(L,LIM)-1000.0-W(L,LIM)
202 CONTINUE
C
C INTERPOLATION OF MACH NUMBER CORRESPONDING TO VARIOUS OMEGA VALUES.
C N AS A CONTROL COUNTER FOR NUMBER OF TRIALS.
DO 206 L=1, 27
A=6.0**0.5
DO 206 K=1, LIM
IF(K.LT.LIM) GO TO 207
M(L,K)=M(L,LIM)
GO TO 209
207 IF(L.EQ.1.AND.K.EQ.1) GO TO 209
N=1
FMS=1.0
210 B=FMS/A
C=A*ATAN(B)
D=ATAN(FMS)
IF(C.GT.D) GO TO 211
C=C+1.570796
211 FMSNEW=FMS-(C-D-W(L,K)*3.14159/180.0)/((1.0+C**2)**(-1)-(1.0+
1FMS**2)**(-1))
IF(ABS(FMS-FMSNEW).LT.1.E-5.OR.N.GT.20) GO TO 212
N=N+1
FMS=FMSNEW
GO TO 210
212 FMSSQ=FMS**2
MS(L,K)=SQRT(6.0*(1.0+FMSSQ)/(6.0+FMSSQ))
SQMS=MS(L,K)**2
SQM=(SQMS/1.2/(1.0-SQMS/6.0))
M(L,K)=SQRT(SQM)
CON=1.0+0.2*SQM
GO TO 208
209 CON=1.0+0.2*M(L,K)**2
208 DP(L,K)=CON**(-3.5)
DD(L,K)=CON**(-2.5)
DT(L,K)=CON**(-1)
206 CONTINUE
C COMPUTATION OF THE ANGLES OF MACH LINES OF TWO FAMILIES WITH X-AXIS.
DO 213 L=1, 26
DO 213 K=1, LIM
IF(L.EQ.1.AND.K.EQ.1) GO TO 214
AL(L,K)=ATAN((M(L,K)**2-1.0)**(-0.5))*180.0/3.14159
GO TO 215
214 AL(1,1)=ATAN((M(1,1)**2-1.0)**(-0.5))*180.0/3.14159
O(1,1)=0.0
215 OAL1(L,K)=(O(L,K)+AL(L,K))
213 OAL2(L,K)=(O(L,K)-AL(L,K))

```

C ANGLES OF MACH LINES IN FAMILY 1 WITH X-AXIS.

```

DO 216 L=1,26
DO 216 K=1, LIM
IF(K.GT.1) GO TO 217
IF(K.GT.LIM) GO TO 218
SLOP1(L,K)=0.5*(OAL1(L,K)+OAL1(L,K+1))
SLOP2(L,K)=0.5*(OAL2(L,K)+OAL2(L+1,K-1))
GO TO 216
217 SLOP1(L,1)=0.5*(OAL1(L,1)+OAL1(L,2))
SLOP2(L,1)=0.00
GO TO 216
218 SLOP1(L,LIM)=O(L,LIM)
SLOP2(L,LIM)=0.5*(OAL2(L,LIM)+OAL2(L+1, LIM-1))
216 WRITE(6, 500) L, K, SLOP1(L,K), SLOP2(L,K)
500 FORMAT(5X, 'L=', I2, 5X, 'K=', I2, 5X, 'ANGLE W1=', F7.3, 5X,
1'ANGLE W2=', F7.3)

```

C COORDINATES OF POINTS, (X(J,K),Y(J,K)), OF THE INTERSECTIONS OF
C MACH LINES AND/OR JET AXIS BASED ON X=0,Y=0 AT THE INTERSECTION POINT
C OF EXIT PLANE AND JET AXIS.

C THE RADIUS OF THE NOZZLE IS DENOTED AS 'RN'.

C CALCULATION FOR THE COORDINATES OF POINT (1,1)

```

RN=170.0/2.0
PIR=3.14159/180.0
J=1
K=1

```

```

Y(1,1)=0.0
X(1,1)=RN/TAN(SLOP1(1,1)*PIR)
WRITE(6,1004) J, K, X(J,K), Y(J,K)

```

C CALCULATION FOR THE COORDINATES OF POINTS(1,K), WHERE K=2, LIM.

```

DO 225 K=2, LIM
TANS1=TAN(SLOP1(1,K)*PIR)
CSLOP2=SLOP2(1,K)+90.0
IF(ABS(CSLOP2).LT.0.1) GO TO 222
IF(ABS(SLOP2(1,K)).GT.90.0) GO TO 223
TANS2=TAN(SLOP2(1,K)*PIR)
GO TO 224
222 X(1,K)=X(1,K-1)
Y(1,K)=X(1,K-1)*TAN(SLOP1(1,K)*PIR)-RN
GO TO 225
223 TANS2=1.0/TAN((-SLOP2(1,K)-90.0)*PIR)
224 X(1,K)=(RN+Y(1,K-1)-X(1,K-1)*TANS2)/(TANS1-TANS2)
Y(1,K)=-RN+(RN+Y(1,K-1)-X(1,K-1)*TANS2)*TANS1/(TANS1-TANS2)
WRITE(6,1004) J, K, X(J,K), Y(J,K)

```

```

225 CONTINUE

```

C CALCULATION FOR THE COORDINATES OF POINTS(J,K), WHERE J=2, 25, AND K=1, LIM.

```

DO 227 J=2, 25
DO 227 K=1, LIM
IF(K.GT.1) GO TO 226
TANS1=TAN(SLOP1(J,1)*PIR)
Y(J,1)=0.0
X(J,1)=-Y(J-1,2)/TANS1+X(J-1,2)
GO TO 228
226 TANS1=TAN(SLOP1(J,K)*PIR)
TANS2=TAN(SLOP2(J,K)*PIR)
IF(K.EQ.LIM) GO TO 229
X(J,K)=(Y(J,K-1)-Y(J-1,K+1)+X(J-1,K+1)*TANS1-X(J,K-1)*TANS2)/
1(TANS1-TANS2)
Y(J,K)=Y(J-1,K+1)+(Y(J,K-1)-Y(J-1,K+1)-(X(J,K-1)-X(J-1,K+1))*TANS2
1)/(TANS1-TANS2)*TANS1

```

```

      GO TO 228
229  X(J,K)=(Y(J,K-1)-Y(J-1,K)+X(J-1,K)*TANS1-X(J,K-1)*TANS2)/
      1(TANS1-TANS2)
      Y(J,K)=Y(J-1,K)+(Y(J,K-1)-Y(J-1,K)-(X(J,K-1)-X(J-1,K))*TANS2)
      1/(TANS1-TANS2)*TANS1
228  WRITE(6,1004) J, K, X(J,K), Y(J,K)
227  CONTINUE
1004  FORMAT(/5X, 'J=', I2, 5X, 'K=', I2, 5X, 'X=', F10.2, 5X, 'Y=',
      1F10.2)
      DO 700 I=1, 25
      DO 700 J=1,50
700  WRITE(1,1500) M(I,J), O(I,J), DP(I,J), DT(I,J), X(I,J), Y(I,J),
      1SLOP1(I,J), SLOP2(I,J), I, J
1500  FORMAT(F7.3, F8.3, F8.4, F8.4, F10.2, F10.2, F8.3, F8.3,
      1 6X, I2, 2X, I2)
      GO TO 50
      END
'XQT
+1.01      +(-.00      +250.00      +1.00
'FIN

```

LIST OF SYMBOLS

The Supersonic Underexpanded Air Jet

AL	α in (degree)
DD	ρ/ρ_0
DP	P/P_0
DT	T/T_0
I1 and I2	K_I and K_{II} in Eq. (30) respectively
M	Mach number, M
MS	M^*
θ	θ in (degree)
θ_{AL1}	$(\theta + \alpha)$ in (degree)
θ_{AL2}	$(\theta - \alpha)$ in (degree)
PE	P_e in (mmHg)
PEP θ	P_e/P_0
P θ	P_0 in (mmHg)
SL θ P1, SL θ P2	$(\theta \pm \alpha)$ in (degree)
W	$\omega(M^*)$ in Eq. (31)
X	x in . Cartesian coordinates
Y	y in . Cartesian coordinates

APPENDIX C**COMPUTER PROGRAM FOR PARTICLE TRAJECTORY
IN THE SUPERSONIC UNDEREXPANDED AIR JET**

```
'RUN PROJ1,304-02-120,WHANG,2,600
```

```
'ASG.T TAPE,8C,4576N
```

```
'DELETE,C FLOF.
```

```
'ASG.UP FLOF,F
```

```
'COPIN TAPE.,FLOF.
```

```
'FOR.IS MAIN
```

```
C
```

```
C CALCULATION FOR PARTICLE TRAJECTORY WITH KNOWN INITIAL POSITION OF PARTICLE,  
C X(0)=0.00, Y(0)=-PP, BETA(0)=0.00.
```

```
C THE COORDINATES OF POINT OF INTERSECTION OF PARTICLE
```

```
C PATH WITH FIRST MACH LINE.
```

```
    DIMENSION M(25,81), THETA(25,81), X(25,81), Y(25,81),  
    1 SLOP1(25,81), SLOP2(25,81), DP(25,81), DT(25,81)
```

```
    DIMENSION XP(100), YP(100), BETA(100), VP(100), VPX(100), VPY(100)  
    DIMENSION PPA(50), DIAPA(50), ANGLEA(50)
```

```
    REAL M
```

```
C IN FLOW FIELD (1,1)
```

```
C
```

```
    DENSO=1.1845
```

```
    PI=180.0/3.14159
```

```
    PIR=3.14159/180.0
```

```
    READ(5,4998) GARMA, TEMPO, RN, PO
```

```
4998 FORMAT(4F10.0)
```

```
    WRITE(6,4997) GARMA, TEMPO, RN, PO
```

```
4997 FORMAT(///5X, 'GARMA=', F5.2/, 5X, 'TEMPERATURE(0)=', F7.2/,  
    1 5X, 'RADIUS OF NOZZLE EXIT=', F7.2/, 5X, 'PRESSURE(0)=', F7.2///)
```

```
    DO 4999 IN=1,42,1
```

```
4999 READ(5,5000) PPA(IN), DIAPA(IN), ANGLEA(IN)
```

```
5000 FORMAT(3F10.0)
```

```
    INPUT=1
```

```
    DO 10 JJ=1,24
```

```
    DO 10 KK=1,47,1
```

```
    10 READ(5, 105) M(JJ, KK), THETA(JJ, KK), DP(JJ, KK), DT(JJ, KK),  
    1 X(JJ, KK), Y(JJ, KK), SLOP1(JJ, KK), SLOP2(JJ, KK)
```

```
105 FORMAT(F7.3, F8.3, F8.4, F8.4, F10.2, F10.2, F8.3, F8.3)
```

```
    DO 11 JJ=1,24,1
```

```
    DO 11 KK=1,47,1
```

```
    X(JJ, KK)=X(JJ, KK)/85.0*RN
```

```
    11 Y(JJ, KK)=Y(JJ, KK)/85.0*RN
```

```
5001 RN=RN
```

```
    PP=PPA(INPUT)
```

```
    DIAP=DIAPA(INPUT)
```

```
    ANGLE=ANGLEA(INPUT)
```

```
    WRITE(6,5002) INPUT, PP, DIAP, ANGLE
```

```
5002 FORMAT(///5X, 'INPUT=', I2, 5X, 'POSITION OF PART.=', F8.4, 5X,  
    1 'DIA. OF PARTICLE=', F7.4, 5X, 'ANGLE=', F8.4, ///)
```

```
    J=1
```

```
    K=1
```

```
    L=1
```

```
    VPX(L)=31600.0*COS(ANGLE)
```

```
    VPY(L)=31600.0*SIN(ANGLE)
```

```
    BETA(L)=ANGLE*PI
```

```
    YP(L)=-PP
```

```
    XP(L)=(YP(L)+RN)*X(J, K)/(Y(J, K)+RN)
```

```
    RENOX=0.00
```

```
    RENYOY=0.00
```

```
    RENO=0.00
```

```
    WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,  
    1 RENYOY, RENO
```

```

99 FORMAT(5X, 'L=', I2, 2X, 'XP=', F8.1, 2X, 'YP=', F8.1, 2X,
1'BETA=', F8.4, 2X, 'VPX=', F8.1, 2X, 'VPY=', F8.1, 2X, 'RENOX=',
2E8.3, 2X, 'RENOY=', E8.3, 2X, 'RENO=', E8.3)
L=L+1
J=J
K=K+1
VP(L-1)=31600.0
BETA(L-1)=ANGLE*PI
VP(L)=31600.0
BETA(L)=ANGLE*PI
RENOX=0.00
RENOY=0.00
RENO=0.00
XP(L)=XP(L-1)
YP(L)=-PP
WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,
1 RENOY, RENO
C
C IN FLOW FIELD (1,2)
350 ABETA=0.5*(BETA(L-1)+BETA(L))
TANBE=TAN(ABETA*PI)
XPNEW=((Y(J,K)-YP(L-1))*X(J,K)+XP(L-1)*X(J,K)*TANBE-X(J,K)*(RN+
1Y(J,K)))/(X(J,K)*TANBE-RN-Y(J,K))
YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)
IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(Y,NEW-YP(L)).LE.0.005) GO TO
1 354
A=ABS(RN)
B=+).00
C=ABS(Y(J,K))
D=ABS(X(J,K))
ABSXP=ABS(XPNEW)
ABSYP=ABS(YPNEW)
IF(ABS(YPNEW-Y(J,K)).LE.0.02.AND.ABS(XPNEW-X(J,K)).LE.0.02) GO
1TO 352
IF(ABSYP.LT.A.AND.ABSYP.GT.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1 GO TO 352
TANBE=TAN(ABETA*PI)
XJK=X(J,K)-X(J,K-1)
YJK=Y(J,K)-Y(J,K-1)
XPNEW=(XJK*(XP(L-1)*TANBE+Y(J,K-1)-YP(L-1))-YJK*X(J,K-1))/
1(XJK*TANBE-YJK)
YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)
IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(YPNEW-YP(L)).LE.0.005) GO TO
1 354
352 XP(L)=XPNEW
YP(L)=YPNEW
C
C FIND THE DELTA LENGTH ALONG WHICH PARTICLE TRAVELS THROUGH FLOW
C FIELD NO.(1,K)
DELLX=XP(L)-XP(L-1)
C IN ORDER TO AVOID DIVIDE-OVERFLOW, ASSIGN DT(J,K)=0.01, WHEN DT(J,K) IS LESS
C THAN 0.01 AND DP(J,K)=0.0001 FOR DP(J,K) LESS THAN 0.0001.
IF(DT(J,K).LT.0.01) DT(J,K)=0.01
TEMP=DT(J,K)*TEMPO
CALL AIR(TEMP, ETA)
C SCALE FACTOR FOR TAU IS -5
TAU=(1.0/18.0)*(DIAP**2.0)*10.0**(+3.0)/ETA
CALL CORR(DIAP, DP(J,K), DT(J,K), PO, TEMPO, CUNM)
C SCALE FACTOR FOR E IS +5

```

```
'RUN PROJ1,304-02-120,WHANG,2,600
```

```
'ASG,T TAPE,8C,4576N
```

```
'DELETE,C FLOF.
```

```
'ASG,UP FLOF,F
```

```
'CDPIN TAPE.,FLOF.
```

```
'FOR,IS MAIL
```

```
C
```

```
C CALCULATION FOR PARTICLE TRAJECTORY WITH KNOWN INITIAL POSITION OF PARTICLE,  
C X(0)=0.00, Y(0)=-PP, BETA(0)=0.00.
```

```
C THE COORDINATES OF POINT OF INTERSECTION OF PARTICLE
```

```
C PATH WITH FIRST MACH LINE.
```

```
    DIMENSION M(25,81), THETA(25,81), X(25,81), Y(25,81),
```

```
    1 SLOP1(25,81), SLOP2(25,81), DP(25,81), DT(25,81)
```

```
    DIMENSION XP(100), YP(100), BETA(100), VP(100), VPX(100), VPY(100)
```

```
    DIMENSION PPA(50), DIAPA(50), ANGLEA(50)
```

```
    REAL M
```

```
C IN FLOW FIELD (1,1)
```

```
C
```

```
    DENSO=1.1845
```

```
    PI=180.0/3.14159
```

```
    PIR=3.14159/180.0
```

```
    READ(5,4998) GARMA, TEMPO, RN, PO
```

```
4998 FORMAT(4F10.0)
```

```
    WRITE(6,4997) GARMA, TEMPO, RN, PO
```

```
4997 FORMAT(///5X, 'GARMA=', F5.2/, 5X, 'TEMPERATURE(0)=', F7.2/,
```

```
    1 5X, 'RADIUS OF NOZZLE EXIT=', F7.2/, 5X, 'PRESSURE(0)=', F7.2///)
```

```
    DO 4999 IN=1,42,1
```

```
4999 READ(5,5000) PPA(IN), DIAPA(IN), ANGLEA(IN)
```

```
5000 FORMAT(3F10.0)
```

```
    INPUT=1
```

```
    DO 10 JJ=1,24
```

```
    DO 10 KK=1,47,1
```

```
    10 READ(5, 105) M(JJ, KK), THETA(JJ, KK), DP(JJ, KK), DT(JJ, KK),
```

```
    1 X(JJ, KK), Y(JJ, KK), SLOP1(JJ, KK), SLOP2(JJ, KK)
```

```
105 FORMAT(F7.3, F8.3, F8.4, F8.4, F10.2, F10.2, F8.3, F8.3)
```

```
    DO 11 JJ=1,24,1
```

```
    DO 11 KK=1,47,1
```

```
    X(JJ, KK)=X(JJ, KK)/85.0*RN
```

```
    11 Y(JJ, KK)=Y(JJ, KK)/85.0*RN
```

```
5001 RN=RN
```

```
    PP=PPA(INPUT)
```

```
    DIAP=DIAPA(INPUT)
```

```
    ANGLE=ANGLEA(INPUT)
```

```
    WRITE(6,5002) INPUT, PP, DIAP, ANGLE
```

```
5002 FORMAT(///5X, 'INPUT=', I2, 5X, 'POSITION OF PART.=', F8.4, 5X,
```

```
    1 'DIA. OF PARTICLE=', F7.4, 5X, 'ANGLE=', F8.4, ///)
```

```
    J=1
```

```
    K=1
```

```
    L=1
```

```
    VPX(L)=31600.0*COS(ANGLE)
```

```
    VPY(L)=31600.0*SIN(ANGLE)
```

```
    BETA(L)=ANGLE*PI
```

```
    YP(L)=-PP
```

```
    XP(L)=(YP(L)+RN)*X(J, K)/(Y(J, K)+RN)
```

```
    RENOX=0.00
```

```
    RENY=0.00
```

```
    RENO=0.00
```

```
    WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,  
    1RENOY, RENO
```

```

99 FORMAT(5X, 'L=', 12, 2X, 'XP=', F8.1, 2X, 'YP=', F8.1, 2X,
1'BETA=', F8.4, 2X, 'VPX=', F8.1, 2X, 'VPY=', F8.1, 2X, 'RENOX=',
2E8.3, 2X, 'RENOY=', E8.3, 2X, 'RENO=', E8.3)

```

```

L=L+1

```

```

J=J

```

```

K=K+1

```

```

VP(L-1)=31600.0

```

```

BETA(L-1)=ANGLE*PI

```

```

VP(L)=31600.0

```

```

BETA(L)=ANGLE*PI

```

```

RENOX=0.00

```

```

RENOY=0.00

```

```

RENO=0.00

```

```

XP(L)=XP(L-1)

```

```

YP(L)=-PP

```

```

WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,
1 RENOY, RENO

```

```

C

```

```

C IN FLOW FIELD (1,2)

```

```

350 ABETA=0.5*(BETA(L-1)+BETA(L))

```

```

TANBE=TAN(ABETA*PI)

```

```

XPNEW=((Y(J,K)-YP(L-1))*X(J,K)+XP(L-1)*X(J,K)*TANBE-X(J,K)*(RN+
1Y(J,K)))/(X(J,K)*TANBE-RN-Y(J,K))

```

```

YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)

```

```

IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(YPNEW-YP(L)).LE.0.005) GO TO
1 354

```

```

A=ABS(RN)

```

```

B=+0.00

```

```

C=ABS(Y(J,K))

```

```

D=ABS(X(J,K))

```

```

ABSXP=ABS(XPNEW)

```

```

ABSY=ABS(YPNEW)

```

```

IF(ABS(YPNEW-Y(J,K)).LE.0.02.AND.ABS(XPNEW-X(J,K)).LE.0.02) GO
1 TO 352

```

```

IF(ABSY.LT.A.AND.ABSY.GT.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1 GO TO 352

```

```

TANBE=TAN(ABETA*PI)

```

```

XJK=X(J,K)-X(J,K-1)

```

```

YJK=Y(J,K)-Y(J,K-1)

```

```

XPNEW=(XJK*(XP(L-1)*TANBE+Y(J,K-1)-YP(L-1))-YJK*X(J,K-1))/
1(XJK*TANBE-YJK)

```

```

YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)

```

```

IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(YPNEW-YP(L)).LE.0.005) GO TO
1 354

```

```

352 XP(L)=XPNEW

```

```

YP(L)=YPNEW

```

```

C

```

```

C FIND THE DELTA LENGTH ALONG WHICH PARTICLE TRAVELS THROUGH FLOW
C FIELD NO.(1,K)

```

```

DELLX=XP(L)-XP(L-1)

```

```

C IN ORDER TO AVOID DIVIDE-OVERFLOW, ASSIGN DT(J,K)=0.01, WHEN DT(J,K) IS LESS
C THAN 0.01 AND DP(J,K)=0.0001 FOR DP(J,K) LESS THAN 0.0001.

```

```

IF(DT(J,K).LT.0.01) DT(J,K)=0.01

```

```

TEMP=DT(J,K)*TEMPO

```

```

CALL AIR(TEMP, ETA)

```

```

C SCALE FACTOR FOR TAU IS -5

```

```

TAU=(1.0/18.0)*(DIAP**2.0)*10.0**(+3.0)/ETA

```

```

CALL (ORR(DIAP, DP(J,K), DT(J,K), PO, TEMPO, CUNM)

```

```

C SCALE FACTOR FOR E IS +5

```

```

E=1.0/(TAU*CUNM)
CALL SOUND(GARMA, TEMP, SUND)
VAIR=M(J,K)*SUND
VAIRX=VAIR*COS(THETA(J,K)*PIR)
VAIRY=VAIR*SIN(THETA(J,K)*PIR)
VPX(L-1)=VP(L-1)*COS(BETA(L-1)*PIR)
VPY(L-1)=VP(L-1)*SIN(BETA(L-1)*PIR)
VPX(L)=VP(L)*COS(BETA(L)*PIR)
VPY(L)=VP(L)*SIN(BETA(L)*PIR)
AVPX=0.5*(VPX(L-1)+VPX(L))
AVPY=0.5*(VPY(L-1)+VPY(L))
C SCALE FACTOR FOR DEL IS -5
DELT=DELLX/(AVPX*10.0**(-1))
C SCALE FACTORS FOR DELVPX AND DELVPY ARE 1.0.
DELVPX=E*(VAIRX-AVPX)*DELT
DELVPY=E*(VAIRY-AVPY)*DELT
VPX(L)=VPX(L-1)+DELVPX
VPY(L)=VPY(L-1)+DELVPY
CON=1.0+0.2*M(J,K)**2
DD=CON**(-3.5)
RENOX=DIAP*DD*DENS0*(VPX(L)-VAIRX)/ETA*0.1
RENOY=DIAP*DD*DENS0*(VPY(L)-VAIRY)/ETA*0.1
RENO=SQRT(RENOX**2.0+RENOY**2.0)
BETAN=ATAN(VPY(L)/VPX(L))*PI
VP(L)=SQRT((VPX(L)/100.0)**(2.0)+(VPY(L)/100.0)**(2.0))*100.0
BETA(L)=BETAN
WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,
1 RENOY, RENO
GO TO 350
354 XP(L)=XPNEW
YP(L)=YPNEW
ABSXP=ABS(XP(L))
ABSY=ABS(YP(L))
YPLYJK=ABS(YP(L)-Y(J,K))
XPLXJK=ABS(XP(L)-X(J,K))
IF(YPLYJK.LE.0.02.AND.XPLXJK.LE.0.02) GO TO 358
IF(SLOP1(J,K).GE.0.00) GO TO 355
IF(ABSY.GT.A.AND.ABSY.LT.C.AND.ABSXP.GT.E.AND.ABSXP.LT.D)
1 GO TO 356
355 IF(ABSY.LE.A.AND.ABSY.GE.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1 GO TO 356
GO TO 360
356 J=J
K=K+1
L=L+1
VP(L)=VP(L-1)
VPX(L)=VPX(L-1)
VPY(L)=VPY(L-1)
BETA(L)=BETA(L-1)
WRITE(6, 199) J, K, L
199 FORMAT(5X, 'J=', I2, 2X, 'K=', I2, 2X, 'L=', I2)
IF(J.GT.20) GO TO 366
GO TO 362
C
358 J=J+1
K=K
L=L+1
IF(K.LE.0) GO TO 359
VP(L)=VP(L-1)

```

```

VPX(L)=VPX(L-1)
VPY(L)=VPY(L-1)
BETA(L)=BETA(L-1)
WRITE(6,199) J, K, L
IF(J.GT.20) GO TO 366
GO TO 362

```

```

359 J=J+1

```

```

K=1
L=L+1
VP(L)=VP(L-1)
VPX(L)=VPX(L-1)
VPY(L)=-VPY(L-1)
BETA(L)=-BETA(L-1)
WRITE(6,199) J, K, L
IF(J.GT.20) GO TO 366
GO TO 362

```

C

```

360 J=J+1

```

```

K=K-1
L=L+1
IF(K.LE.0) GO TO 361
VP(L)=VP(L-1)
VPX(L)=VPX(L-1)
VPY(L)=VPY(L-1)
BETA(L)=BETA(L-1)
WRITE(6,199) J, K, L
IF(J.GT.20) GO TO 366
GO TO 362

```

```

361 J=J+1

```

```

K=1
L=L+1
VP(L)=VP(L-1)
VPX(L)=VPX(L-1)
VPY(L)=-VPY(L-1)
BETA(L)=-BETA(L-1)
WRITE(6,199) J, K, L
IF(J.GT.20) GO TO 366
GO TO 362

```

```

362 ABETA=0.5*(BETA(L)+BETA(L-1))

```

```

TANBE=TAN(ABETA*PIR)
IF(J.(Q.1) GO TO 365
XJK=X(J-1,K+1)-X(J,K)
YJK=Y(J-1,K+1)-Y(J,K)
XPNEW=(XJK*(Y(J,K)-YP(L-1)+XP(L-1)*TANBE)-X(J,K)*YJK)/(XJK*TANBE
1-YJK)

```

```

YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)

```

```

XPNXPL=ABS(XPNEW-XP(L))

```

```

YPNYPL=ABS(YPNEW-YP(L))

```

```

A=ABS(Y(J-1,K+1))

```

```

B=ABS(X(J-1,K+1))

```

```

C=ABS(Y(J,K))

```

```

D=ABS(X(J,K))

```

```

ABSXP=ABS(XPNEW)

```

```

ABSYPL=ABS(YPNEW)

```

```

IF(XPNXPL.LE.0.005.AND.YPNYPL.LE.0.005) GO TO 354

```

```

YPNYJK=ABS(YPNEW-Y(J,K))

```

```

XPXJK=ABS(XPNEW-X(J,K))

```

```

IF(YPNYJK.LT.0.02.AND.XPNXJK.LT.0.02) GO TO 364

```

```

ABSYPL=ABS(YPNEW)

```

```

ABSXP=ABS(XPNEW)
A=ABS(Y(J-1,K+1))
B=ABS(X(J-1,K+1))
C=ABS(Y(J,K))
D=ABS(X(J,K))
IF(SLOP1(J,K).LE.0.00) GO TO 363
IF(ABSYP.LT.A.AND.ABSYP.GT.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1 GO TO 364
363 IF(ABSYP.GE.A.AND.ABSYP.LE.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1GO TO 364
TANBE=TAN(ABETA*PIR)
IF(K.EQ.1) GO TO 400
XJK=X(J,K-1)-X(J,K)
YJK=Y(J,K-1)-Y(J,K)
XPNEW=(XJK*(Y(J,K)-YP(L-1)+XP(L-1)*TANBE)-Y(J,K)*YJK)/(XJK*TANBE
1-YJK)
YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)
GO TO 401
400 XJK=X(J-1,K)-X(J,K)
Y(J,K)=0.00
YJK=0.00
YPNEW=0.00
XPNEW=(XJK*(Y(J,K)-YP(L-1)+XP(L-1)*TANBE)-(J,K)*YJK)/(XJK*TANBE
1-YJK)
401 XPNXPL=ABS(XPNEW-XP(L))
YPNYPL=ABS(YPNEW-YP(L))
IF(XPNXPL.LE.0.005.AND.YPNYPL.LE.0.005) GO TO 354
GO TO 364
365 XPNEW=((Y(J,K)-YP(L-1))*X(J,K)+XP(L-1)*X(J,K)*TANBE-X(J,K)*(RN+
1Y(J,K)))/(X(J,K)*TANBE-RN-Y(J,K))
YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)
A=ABS(RN)
B=+0.00
C=ABS(Y(J,K))
D=ABS(X(J,K))
ABSXP=ABS(XPNEW)
ABSYP=ABS(YPNEW)
IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(YPNEW-YP(L)).LE.0.005) GO TO
1 354
A=ABS(RN)
B=+0.00
C=ABS(Y(J,K))
D=ABS(X(J,K))
ABSXP=ABS(XPNEW)
ABSYP=ABS(YPNEW)
IF(ABS(YPNEW-Y(J,K)).LE.0.02.AND.ABS(XPNEW-X(J,K)).LE.0.02) GO
1TO 364
IF(ABSYP.LT.A.AND.ABSYP.GT.C.AND.ABSXP.GT.B.AND.ABSXP.LT.D)
1 GO TO 364
TANBE=TAN(ABETA*PIR)
XJK=X(J,K)-X(J,K-1)
YJK=Y(J,K)-Y(J,K-1)
XPNEW=(XJK*(XP(L-1)*TANBE+Y(J,K-1)-YP(L-1))-YJK*X(J,K-1))/
1(XJK*TANBE-YJK)
YPNEW=(XPNEW-XP(L-1))*TANBE+YP(L-1)
IF(ABS(XPNEW-XP(L)).LE.0.005.AND.ABS(YPNEW-YP(L)).LE.0.005) GO TO
1 354
364 XP(L)=XPNEW
YP(L)=YPNEW

```

```

DELLX=XP(L)-XP(L-1)
TEMP=TEMPO*DT(J,K)

C
CALL AIR(TEMP, ETA)

C
TAU=(1.0/18.0)*(DIAP**2)/ETA*10.0**(4)
CALL CORR(DIAP, DP(J,K), DT(J,K), PO, TEMPO, CUNM)
E=1.0/(TAU*CUNM)

C
CALL SOUND(GARMA, TEMP, SUND)
VAIR=M(J,K)*SUND
VAIRX=VAIR*COS(THETA(J,K)*PIR)
VAIRY=VAIR*SIN(THETA(J,K)*PIR)
VPX(L-1)=VP(L-1)*COS(BETA(L-1)*PIR)
VPY(L-1)=VP(L-1)*SIN(BETA(L-1)*PIR)
VPX(L)=VP(L)*COS(BETA(L)*PIR)
VPY(L)=VP(L)*SIN(BETA(L)*PIR)
AVPX=0.5*(VPX(L-1)+VPX(L))
AVPY=0.5*(VPY(L-1)+VPY(L))
DELT=DELLX/(VPX(L-1)*10.0**(-1))
DELVPX=E*(VAIRX-AVPX)*DELT
DELVPY=E*(VAIRY-AVPY)*DELT
VPX(L)=VPX(L-1)+DELVPX
VPY(L)=VPY(L-1)+DELVPY
CON=1.0+0.2*M(J,K)**2
DD=CON**(-3.5)
RENOX=DIAP*DD*DENS0*(VPX(L)-VAIRX)/ETA*0.1
RENOY=DIAP*DD*DENS0*(VPY(L)-VAIRY)/ETA*0.1
RENO=SQRT(RENOX**2.0+RENOY**2.0)
BETAN=ATAN(VPY(L)/VPX(L))*PI
VP(L)=SQRT((VPX(L)/100.0)**(2.0)+(VPY(L)/100.0)**(2.0))*100.0
BETA(L)=BETAN
WRITE(6,99) L, XP(L), YP(L), BETA(L), VPX(L), VPY(L), RENOX,
1 RENOY, RENO
GO TO 362
366 LIM=L
C DNT DENOTES THE DISTANCE BETWEEN NOZZLE AND TARGET.
DNT=5.0*10.0**3
C DDNT DENOTES DISTANCE BETWEEN TARGET AND FINAL EXIT OF PARTICLE FROM LAST FLOW
C FIELD
DDNT=DNT-XP(L-1)
XP(L)=DNT
YP(L)=YP(L-1)+VPY(L-1)/VPX(L-1)*DDNT
DO 368 L=1,LIM
368 WRITE(6,1005) L, VPX(L), VPY(L), XP(L), YP(L), BETA(L)
1005 FORMAT(5X, 'L=', I2, 5X, 'VPX=', F10.4, 5X, 'VPY=', F10.4, 5X,
1 'X=', F9.3, 5X, 'Y=', F9.3, 5X, 'BETA=', F7.2)
INPUT=INPUT+1
IF(INPUT.GT.42) GO TO 5100
GO TO 5001
5100 END
*FOR,IS AIR
SUBROUTINE AIR(TEMP, ETA)
C SUBROUTINE NO.1 FOR AIR VISCOSITY, 'ETA', AT DIFFERENT ABSOLUTE TEMPERATURE,
C 'TEMP', IN DEGREE KELVIN
C VISCOSITY IS IN MICROPOISE
DIMENSION TTEMP(9), TETA(9)
TTEMP(1)=312.60
TTEMP(2)=+219.0

```

```

TTEMP(3)=+272.6
TTEMP(4)=+241.0
TTEMP(5)=+203.2
TTEMP(6)=+168.6
TTEMP(7)=+89.7
TTEMP(8)=+78.4
TTEMP(9)=0.00
TETA(1)=190.4
TETA(2)=182.7
TETA(3)=170.8
TETA(4)=153.9
TETA(5)=133.3
TETA(6)=113.0
TETA(7)=62.7
TETA(8)=55.1
TETA(9)=0.00
IF(TEMP.LE.312.6.AND.TEMP.GT.290.6) GO TO 501
IF(TEMP.LE.290.6.AND.TEMP.GT.272.6) GO TO 502
IF(TEMP.LE.272.6.AND.TEMP.GT.241.0) GO TO 503
IF(TEMP.LE.241.0.AND.TEMP.GT.203.2) GO TO 504
IF(TEMP.LE.203.2.AND.TEMP.GT.168.6) GO TO 505
IF(TEMP.LE.168.6.AND.TEMP.GT.89.5) GO TO 506
IF(TEMP.LE.89.5.AND.TEMP.GT.78.4) GO TO 507
IF(TEMP.LE.78.4.AND.TEMP.GT.0.00) GO TO 508
ETA=0.00
GO TO 510
508 J=8
GO TO 510
507 J=7
GO TO 510
506 J=6
GO TO 510
505 J=5
GO TO 510
504 J=4
GO TO 510
503 J=3
GO TO 510
502 J=2
GO TO 510
501 J=1
C SCALE FACTOR FOR VISCOSITY , ETA, IS -6
510 ETA=((TEMP-TTEMP(J+1))/(TTEMP(J)-TTEMP(J+1))*(TETA(J)-TETA(J+1))
1+TETA(J+1))
RETURN
END
*FOR,IS CORR
SUBROUTINE CORR(DIAP, DPSUB, DTSUB, PO, TEMPO, CUNM)
C SUBROUTINE NO. 2 FOR CUNNINGHAM CORRECTION FACTOR AT DIFFERENT ABSOLUTE
C TEMPERATURE AND PRESSURE FOR A GIVEN PARTICLE SIZE
C IN ORDER TO PREVENT DIVIDE-OVERFLOW, ASSIGN DTSUB=0.01 FOR DTSUB LESS THAN
C 0.01 AND DPSUB=0.0001 FOR DPSUB LESS THAN 0.0001.
IF(DTSUB.LT.0.01) DTSUB=0.01
TEMP=TEMPO*DTSUB
IF(DPSUB.LT.0.0001) DPSUB=0.0001
P=PO*DPSUB
C SCALE FACTOR FOR LAMDA IS -5
LAMDA=1.75*TEMP/P
C SCALE FACTOR FOR FLAMDA IS 1.0

```

```

FLAMDA=LAMDA/DIAP/10.0
CUNM=1.0+2.0*FLAMDA*(1.2+0.4*EXP(-0.45/FLAMDA))
WRITE(6,599) LAMDA, CUNM
599 FORMAT(5X, 'LAMDA=', F9.2, 5X, 'CUNM=', F5.2)
RETURN
END
'FOR,IS SOUND
SUBROUTINE SOUND(GARMA, TEMP, SUND)
C SUBROUTINE NO.3 FOR LOCAL VELOCITY OF SOUND AT DIFFERENT ABSOLUTE
C TEMPERATURE AND GARMA
C SCALE FACTOR FOR GASCON IS +3
GASCON=82.0*1.013
WTMO=28.8
CON=GARMA*GASCON/WTMO
SUND=(CON*TEMP)**(0.5)*10.0**3
RETURN
END
'MAP
'XQT
+280.0      +0.365      +0.0815
'ADD FLOF.P30
'FIN

```

```

FLAMDA=LAMDA/DIAP/10.0
CUNM=1.0+2.0*FLAMDA*(1.2+0.4*EXP(-0.45/FLAMDA))
WRITE(6,599) LAMDA, CUNM
599 FORMAT(5X, 'LAMDA=', F9.2, 5X, 'CUNM=', F5.2)
RETURN
END

```

```

'FOR,IS SOUND

```

```

SUBROUTINE SOUND(GARMA, TEMP, SUND)

```

```

C SUBROUTINE NO.3 FOR LOCAL VELOCITY OF SOUND AT DIFFERENT ABSOLUTE
C TEMPERATURE AND GARMA

```

```

C SCALE FACTOR FOR GASCON IS +3

```

```

GASCON=82.0*1.013

```

```

WTMO=;8.8

```

```

CON=GARMA*GASCON/WTMO

```

```

SUND=(CON*TEMP)**(0.5)*10.0**3

```

```

RETURN

```

```

END

```

```

'MAP

```

```

'XQT

```

+1.40	+298.00	+300.0	+30.0
+20.00	+0.500	+0.0068	
+40.0	+0.500	+0.0126	
+60.00	+0.500	+0.0180	
+80.00	+0.500	+0.0234	
+100.0	+0.500	+0.0290	
+120.0	+0.500	+0.0346	
+140.0	+0.500	+0.0402	
+160.0	+0.500	+0.0458	
+180.0	+0.500	+0.0517	
+200.0	+0.500	+0.0576	
+220.0	+0.500	+0.0634	
+240.0	+0.500	+0.0692	
+260.0	+0.500	+0.0757	
+280.0	+0.500	+0.0815	
+20.00	+0.714	+0.0068	
+40.0	+0.714	+0.0126	
+60.00	+0.714	+0.0180	
+80.00	+0.714	+0.0234	
+100.0	+0.714	+0.0290	
+120.00	+0.714	+0.0346	
+140.00	+0.714	+0.0402	
+160.00	+0.714	+0.0458	
+180.00	+0.714	+0.0517	
+200.0	+0.714	+0.0576	
+220.00	+0.714	+0.0634	
+240.00	+0.714	+0.0692	
+260.00	+0.714	+0.0757	
+280.0	+0.714	+0.0815	
+20.00	+0.365	+0.0068	
+40.00	+0.365	+0.0126	
+60.00	+0.365	+0.0180	
+80.00	+0.365	+0.0234	
+100.00	+0.365	+0.0290	
+120.0	+0.365	+0.0346	
+140.00	+0.365	+0.0402	
+160.00	+0.365	+0.0458	
+180.00	+0.365	+0.0517	
+200.0	+0.365	+0.0576	
+220.0	+0.365	+0.0634	

```

+240.0      +0.365      +0.0692
+260.0      +0.365      +0.0757
+280.0      +0.365      +0.0815
'ADD FLOF.P30
'FIN

```

A typical output of the computation is presented in Table 8 . The initial position of the particle is $x = 0$; i. e., the nozzle exit plane, and a radial distance, $y = 240 \mu\text{m}$, from the nozzle axis. The source pressure is $P_o = 250 \text{ mmHg}$, background pressure $P_e = 1 \text{ mmHg}$, particle diameter $d_p = 0.365 \mu\text{m}$, temperature at stagnation state $T_o = 298^\circ\text{K}$, and the diameter of the nozzle exit or throat $D^* = 576 \text{ m}$. The flow fields that are crossed by the particle are shown in the second column of Table 8 . Table 8 also indicates the positions of the particle (x_p, y_p), the velocity components of the particle V_{px} and V_{py} , the Cunningham correction factor and the inclination angle, β , of the particle trajectory in respect to the jet axis at different stages during computation.

Table 6

Calculation of Particle Trajectory in the Supersonic air jet with initial Conditions:

$P_0 = 250 \text{ mmHg}$, $P_e = 1 \text{ mmHg}$, $d_p = 0.365 \mu\text{m}$,

$T_0 = 2980 \text{ K}$, $D^* = 576 \mu\text{m}$ (Nozzle A)

Initial position of particle $X=0$, $r_0 = 240 \mu\text{m}$

Calculation Operation No.	Flow Field No. (see Fig. 7 p. 25)	Coordinates of particle position		Inclination of the Trajectory θ (degree)	Cunningham Corr. Factor	Velocity of Particle (cm/sec)	
		x_p (μm)	y_p (μm)			V_{px}	V_{py}
1	(1, 1)	10.0	240.0	0.000	3.30	31600.0	0.0
2 (1st Iter.)	(1, 2)	10.0	240.0	0.000	3.30	31684.5	22.8
2 (2nd Iter.)	(1, 2)	24.2	240.0	0.0298	3.35	31660.9	16.5
3	(1, 3)	30.5	240.0	0.0564	3.35	31702.2	31.2
4	(1, 4)	35.9	240.0	0.0911	3.35	31747.5	50.5
5	(1, 5)	41.0	240.0	0.1347	3.35	31798.0	74.8
6	(1, 6)	46.0	240.0	0.1775	4.21	31842.5	98.6
7	(1, 7)	51.1	240.1	0.2296	4.21	31892.5	127.8
8	(1, 8)	56.3	240.1	0.2919	4.21	31948.0	162.8
9	(1, 9)	61.7	240.1	0.3664	4.21	32010.1	204.7
10 (1st Iter.)	(1, 10)	67.4	240.1	0.4546	4.21	32071.2	254.5
10 (2nd Iter.)	(1, 10)	67.4	240.2	0.4541	4.21	32078.8	254.3
11 (1st Iter.)	(1, 11)	73.5	240.2	0.5580	4.21	32155.9	313.2
11 (2nd Iter.)	(1, 11)	73.5	240.2	0.5574	4.21	32155.4	312.8
12 (1st Iter.)	(1, 12)	80.1	240.3	0.6586	5.08	32226.6	370.7
13 (1st Iter.)	(1, 13)	87.2	240.4	0.7784	5.08	32306.7	370.4
13 (2nd Iter.)	(1, 13)	87.2	240.4	0.7777	5.08	32306.2	370.4
14 (1st Iter.)	(1, 14)	95.1	240.5	0.9195	5.08	32396.8	438.9
14 (2nd Iter.)	(1, 14)	95.1	240.5	0.9186	5.08	32396.2	438.4
15 (1st Iter.)	(1, 15)	103.8	240.6	1.0852	5.08	32498.0	520.0
15 (2nd Iter.)	(1, 15)	103.7	240.6	1.0839	5.08	32497.2	519.4
16 (1st Iter.)	(1, 16)	113.5	240.8	1.2815	5.08	32613.0	615.6
16 (2nd Iter.)	(1, 16)	113.4	240.8	1.2797	5.08	32611.9	614.8
							728.5

Table 8 (Cont.)

Calculation Operation No.	Flow Field No.	Coordinates of particle position		Inclination of the Trajectory β	Cunning- ham Correc. Factor	Velocity of Particle (cm/sec)	
		x_p	y_p			V_{px}	V_{py}
17(1st Iter.)	(1, 17)	124.4	241.1	1.4787	5.95	32723.9	844.7
17(2nd Iter.)	(1, 17)	124.4	241.1	1.4769	5.95	32722.8	843.7
18(1st Iter.)	(1, 18)	136.9	241.4	1.7144	5.95	32851.0	983.3
18(2nd Iter.)	(1, 18)	136.8	241.4	1.7118	5.95	32849.6	981.7
19(1st Iter.)	(1, 19)	151.2	241.9	1.9954	5.95	32996.9	1149.6
19(2nd Iter.)	(1, 19)	151.1	241.9	1.9916	5.95	32994.9	1147.4
20(1st Iter.)	(1, 20)	167.8	242.5	2.3328	5.95	33166.0	1351.1
20(2nd Iter.)	(1, 20)	167.7	242.5	2.3272	5.95	33163.2	1347.7
21(1st Iter.)	(1, 21)	187.3	243.3	2.6849	6.82	33336.1	1563.3
21(2nd Iter.)	(1, 21)	187.1	243.4	2.6784	6.82	33333.0	1559.4
22(1st Iter.)	(1, 22)	207.8	244.3	3.0659	6.82	33512.0	1759.4
22(2nd Iter.)	(1, 22)	207.9	244.4	3.0640	6.82	33513.1	1793.4
23	(2, 21)	210.0	244.5	3.0998	6.82	33515.8	1815.9
24(1st Iter.)	(2, 22)	237.5	246.0	3.5916	6.82	33779.0	2120.2
24(2nd Iter.)	(2, 22)	237.0	246.1	3.5783	6.82	33772.3	2111.9
24(3rd Iter.)	(2, 22)	237.0	246.1	3.5787	6.82	33772.5	2112.2
25(1st Iter.)	(2, 23)	270.1	248.2	4.1097	7.69	34030.6	2445.1
25(2nd Iter.)	(2, 23)	269.4	248.3	4.0934	7.69	34022.6	2434.8
25(3rd Iter.)	(2, 23)	269.5	248.3	4.0936	7.69	34022.8	2035.2
26(1st Iter.)	(2, 24)	310.0	251.2	4.7454	7.69	34328.7	2849.7
26(2nd Iter.)	(2, 24)	308.8	251.3	4.7196	7.69	34316.5	2833.1
26(3rd Iter.)	(2, 24)	308.8	251.3	4.7206	7.69	34317.0	2833.8
27(1st Iter.)	(2, 25)	346.7	254.4	5.3272	7.69	34592.5	3255.6
27(2nd Iter.)	(2, 25)	346.8	254.7	5.3232	7.69	34590.6	3233.0

Table 8 (Cont.)

Calculation Operation No.	Flow Field No.	Coordinates of Particle position		Inclination of the Trajectory θ	Cunning- ham Correc. Factor	Velocity of Particle (cm/sec)	
		x_p	y_p			V_{px}	V_{py}
28(1st Iter.)	(3, 24)	356.7	255.6	5.4524	8.56	34656.2	3308.0
28(2nd Iter.)	(3, 24)	356.6	255.6	5.4513	8.56	34656.2	3307.3
29(1st Iter.)	(4, 24)	409.4	260.6	6.1590	8.56	35003.3	3777.2
29(2nd Iter.)	(4, 24)	409.7	261.0	6.1535	8.56	35000.6	3773.6
30(1st Iter.)	(4, 24)	413.6	261.4	6.1961	9.44	35024.1	3802.4
30(2nd Iter.)	(4, 24)	413.6	261.4	6.1959	9.44	35024.0	3802.3
31(1st Iter.)	(4, 25)	464.4	266.9	6.7713	9.44	35325.4	4194.6
31(2nd Iter.)	(4, 25)	464.6	267.2	6.7676	9.44	35325.4	4192.1
32(1st Iter.)	(5, 24)	480.8	269.1	6.9320	9.44	35423.1	4306.7
32(2nd Iter.)	(5, 24)	480.7	269.1	6.9302	9.44	35422.0	4305.4
33(1st Iter.)	(5, 25)	516.8	273.5	7.2767	10.31	35617.2	4547.8
33(2nd Iter.)	(5, 25)	516.9	273.6	7.2753	10.31	35616.4	4547.0
34(1st Iter.)	(6, 24)	560.1	279.1	7.6501	10.31	35854.8	4816.0
34(2nd Iter.)	(6, 24)	559.2	279.2	7.6403	10.31	35848.5	4808.8
34(3rd Iter.)	(6, 24)	559.3	279.2	7.6405	10.31	35848.6	4809.0
35(1st Iter.)	(6, 25)	568.7	280.5	7.7178	11.18	35895.1	4864.6
35(2nd Iter.)	(6, 25)	568.7	280.5	7.7177	11.18	35895.0	4864.5
36(1st Iter.)	(7, 24)	621.1	287.6	8.0835	12.06	36141.7	5133.1
36(2nd Iter.)	(7, 24)	621.3	287.8	8.0819	12.06	361406	5131.9
37(1st Iter.)	(8, 23)	649.7	291.8	8.2598	12.06	36276.1	5266.1
37(2nd Iter.)	(8, 23)	649.5	291.8	8.2575	12.06	36274.4	5264.4
38(1st Iter.)	(8, 24)	675.5	295.6	8.4149	12.93	36386.6	5382.8
38(2nd Iter.)	(8, 24)	675.5	295.6	8.4146	12.93	36386.4	5382.6
39(1st Iter.)	(9, 23)	732.0	304.0	8.7047	13.81	36619.7	5606.7
39(2nd Iter.)	(9, 23)	732.2	304.1	8.7036	13.81	36618.8	5606.8
40(1st Iter.)	(10, 22)	753.2	307.4	8.7999	13.81	36706.7	5682.4
40(2nd Iter.)	(10, 22)	753.1	307.4	8.7992	13.81	36706.0	5681.9
41(1st Iter.)	(10, 23)	791.6	313.4	8.9745	14.69	36854.2	5820.3
41(2nd Iter.)	(10, 23)	791.9	313.4	8.9741	14.69	36853.9	5820.0

LIST OF SYMBOLS

The Particle Trajectories in the Jet

Main Program:

ANGLE	Initial β value in (degree)
AVPX and AVPY	$\bar{V}_{px}$ and $\bar{V}_{py}$ in (cm/sec) respectively
BETA	β in (degree)
DD	(S/S_0)
DELVPX	(ΔV_{px}) in (cm/sec)
DELVPY	(ΔV_{py}) in (cm/sec)
DIAP	d_p in (μm)
DP	P/P_0
DT	T/T_0
GARMA	k (the specific heat ratio) ≈ 1.4
M	Mach number
PØ	P_0 in (mmHg)
PPA	the ordinate of a particle at the nozzle exit plane, $ y_p = d_j/2$ in (μm)
RENØ	R_e of a particle (dimensionless)
RENØX and RENØY	$(R_e)_x$ and $(R_e)_y$
RN	R_N^* in (μm)
SLØP1 and SLØP2	$(\ominus + \alpha)$ and $(\ominus - \alpha)$ (in degree)
TAU	τ in (sec)
TEMPØ	T_0 in ($^{\circ}K$)
THETA	$\ominus$ in (degree)
VAIR	V_a in (cm/sec)
VAIRX and VAIRY	V_{ax} and V_{ay} in (cm/sec)

VP	V_p in (cm/sec)
VPX and VPY	V_{px} and V_{py} in (cm/sec)
X and Y	x and y in (μ m)
XP and YP	the Cartesian coordinates of a particle
<u>Subroutine air</u>	
TEMP	T in ($^{\circ}$ K)
ETA	η in (poise)
TTEMP	Table of the temperatures of the air
TETA	Table of the viscosities of the air
<u>Subroutine Corr.</u>	
LAMDA	λ in (cm)
CUNM	the Cunningham correction factor, C
<u>Subroutine Sound</u>	
SUND	the speed of sound wave, c, in (cm/sec)
WTMØ	the molecular weight of the air

LIST OF SYMBOLS

<u>Symbol</u>	<u>Explanation</u>
A	Cross-sectional area of the nozzle
A^*	Cross-sectional area of the nozzle throat or exit
c	Local speed of sound
c_o	Speed of sound at stagnation state
c^*	Speed of sound at Mach number one
C	Cunningham correction factor in Stokes formula
d_j	Diameter of the particle beam at nozzle exit plane
d_p	Particle diameter
D_{dep}	Diameter of particle deposit
D^*	Throat diameter of a nozzle
I_i	Particle density on particle deposit in (number of particles per unit area)
I_{20}	Particle density at a dimensionless distance $100(r_i/R_{dep})\% = 20\%$ from the center of a deposit
k	Specific heat ratio of the fluid, $k = 1.4$ for air
K_I, K_{II}	Integration constants of the characteristics curves of two families respectively
l_i	Stop distance of a particle
M	Local Mach number
M_x, M_y	Components of local Mach number in x- and y-directions respectively
M^*	Mach number expressed in terms of the sonic velocity at $M = 1$
P	Local pressure of the fluid
P_o	Source pressure or pressure of fluid at stagnation state
$(P_o)_{min}$	Source pressure corresponding to the occurrence of minimum beam width

<u>Symbol</u>	<u>Explanation</u>
P_e	Background pressure of the fluid
q	Resultant velocity of a fluid for compressible subsonic flow
q_i	Resultant velocity of a fluid for incompressible subsonic flow
Q	Volumetric flow rate of a nozzle
r_j	Radius of particle beam at nozzle exit plane
r_i	Radial distance from the center of particle deposit
r_p	Particle radius
R_B	Radius of a particle beam
R_{dep}	Radius of a particle deposit
R_e	Renolds number
R_g	Gas constant
R_N	Radius of a nozzle
R_N^*	Radius of the nozzle at the exit plane
S_I, S_{II}	Slopes of the Mach lines of the two families respectively
t	Time
T, T_o	Local absolute temperature and absolute temperature at stagnation state respectively
u	Velocity component of fluid for either compressible subsonic flow or supersonic flow, in x-direction
u_i	Velocity component of the fluid in x-direction for incompressible subsonic flow
v	Velocity component of a fluid for either compressible subsonic flow or supersonic flow in y-direction
v_i	Velocity component of a fluid in y-direction for incompressible subsonic flow
V	Resultant velocity of a fluid in supersonic flow
V_a	Velocity of air
V_o	Initial velocity of a particle

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